The ranks of homotopy groups of Kac-Moody groups*

Zhao Xu-an, zhaoxa@bnu.edu.cn
Department of Mathematics, Beijing Normal University
Key Laboratory of Mathematics and Complex Systems
Ministry of Education, China, Beijing 100875
Jin Chunhua, jinch@amss.ac.cn
Academy of Mathematics and Systems Science, Chinese Academy of Sciences
China, Beijing 100190

Abstract

Let A be a Cartan matrix and $G(A)$ be the Kac-Moody group associated to Cartan matrix A. In this paper, we discuss the computation of the rank i_k of homotopy group $\pi_k(G(A))$. For a large class of Kac-Moody groups, we construct Lie algebras with grade from the Poincaré series of their flag manifolds. And we interpret i_{2k} as the dimension of the degree $2k$ homogeneous component of the Lie algebra we constructed. Since the computation of i_{2k-1} is trivial, this gives a combinatorics interpretation of i_k for all $k > 0$.

Keywords: Cartan matrix, Kac-Moody Group, Flag manifold, Rank of homotopy group, Universal enveloping algebra.

MSC(2010): Primary 55Q52, secondly 17C99.

1 Introduction

Let $A = (a_{ij})$ be an $n \times n$ integer matrix satisfying

(1) For each $i, a_{ii} = 2$;

(2) For $i \neq j, a_{ij} \leq 0$;

(3) If $a_{ij} = 0$, then $a_{ji} = 0$,

then A is called a Cartan matrix.

Let h be the real vector space spanned by $\Pi^\vee = \{\alpha_1^\vee, \alpha_2^\vee, \cdots, \alpha_n^\vee\}$, denote the dual basis of $\Pi^\vee$ in the dual vector space h^* by $\{\omega_1, \omega_2, \cdots, \omega_n\}$. That is $\omega_i(\alpha_j^\vee) = \delta_{ij}$ for $1 \leq i, j \leq n$. Let $\Pi = \{\alpha_1, \cdots, \alpha_n\} \subset h^*$ be given by $\langle \alpha_i^\vee, \alpha_j \rangle = a_{ij}$ for all i, j, then $\alpha_i = \sum\limits_{j=1}^{n} a_{ji}\omega_j$. The triple $(h, \Pi, \Pi^\vee)$ is called the realization of Cartan matrix A. Π and $\Pi^\vee$ are called respectively the simple root system and simple coroot system associated to Cartan matrix A.

By the work of Kac[12] and Moody[24], it is well known that for each Cartan matrix A, there is a Lie algebra $g(A)$ associated to A which is called the Kac-Moody Lie algebra.

*The authors are supported by NSFC 11171025

The Kac-Moody Lie algebra $g(A)$ is generated by $\alpha_i^\vee, e_i, f_i, 1 \le i \le n$ over $\mathbb{C}$, with the defining relations:

(1) $[\alpha_i^\vee, \alpha_j^\vee] = 0$;

(2) $[e_i, f_j] = \delta_{ij}\alpha_i^\vee$;

(3) $[\alpha_i^\vee, e_j] = a_{ij}e_j, [\alpha_i^\vee, f_j] = -a_{ij}f_j$;

(4) $\mathrm{ad}(e_i)^{-a_{ij}+1}(e_j) = 0, \mathrm{ad}(f_i)^{-a_{ij}+1}(f_j) = 0$ for $i \neq j$.

For details, see Kac[13] and Kumar[21].

Kac and Peterson constructed the simply connected Kac-Moody group $G(A)$ with Lie algebra $g(A)$ in [14][15][16].

A Cartan matrix A is symmetrizable if there exists an invertible diagonal matrix D and a symmetric matrix B such that $A = DB$. $g(A)$ and $G(A)$ are symmetrizable if the Cartan matrix A is symmetrizable. A Cartan matrix A is indecomposable if A can not be decomposed into the sum $A_1 \oplus A_2$ of two Cartan matrices A_1, A_2. The Kac-Moody Lie algebra or the Kac-Moody group is indecomposable if the Cartan matrix A is indecomposable.

Indecomposable Cartan matrices and their associated Kac-Moody Lie algebras, Kac-Moody groups are divided into three types.

(1) Finite type, when A is positive definite. In this case, $G(A)$ is just the simply connected complex semisimple Lie group with Cartan matrix A.

(2) Affine type, when A is positive semi-definite and has rank $n-1$.

(3) Indefinite type otherwise.

For the Kac-Moody Lie algebra $g(A)$, there is Cartan decomposition $g(A) = h \oplus \sum\limits_{\alpha\in\Delta} g_\alpha$, where h is a Cartan sub-algebra and Δ is the root system. Let $b = h \oplus \sum\limits_{\alpha\in\Delta^+} g_\alpha$ be the Borel sub-algebra, then b corresponds to a Borel subgroup $B(A)$ in the Kac-Moody group $G(A)$. The homogeneous space $F(A) = G(A)/B(A)$ is called the flag manifold. By Kumar[21], $F(A)$ is an ind-variety.

By rational homotopy theory[27] the ranks of homotopy groups of $G(A)$ can be computed from the rational cohomology of $G(A)$. The rational cohomology rings of Kac-Moody groups and their flag manifolds of finite or affine type have been extensively studied by many people. For reference, see Pontrjagin[25], Hopf[8], Borel[2][3][4], Bott and Samelson[5], Bott[6], Milnor and Moore[23] etc. The structure theory of cohomology rings is well established. For the indefinite case, there are some works by Kumar[20], Kac[16], Kostant and Kumar[19] and Kichiloo[18], Zhao and Jin[30]. The fundamental structure and the explicit algorithm to determine the cohomology are founded. But except for the examples in [18][30], there are no concrete computational examples. This paper will work in this direction and give more examples.

By Kichiloo[18] and Kumar[21], it follows that the rational cohomology rings $H^*(G(A))$ and $H^*(F(A))$ are locally finite and generated by countable number of generators.

By the well known theorem of Hopf about the structure of the rational cohomology ring of a Hopf space G, we know $H^*(G)$ is a Hopf algebra and as algebra it is isomorphic to the tensor product of a polynomial algebra $P(V_0)$ and an exterior algebra $\Lambda(V_1)$, where V_0 and V_1 are respectively the set of even and odd degree free generators of $H^*(G)$. Therefore the Poincaré series of the Kac-

Moody group $G(A)$ of form

$$P_{G(A)}(q)=\prod_{k=1}^{\infty}\frac{(1-q^{2k-1})^{i_{2k-1}}}{(1-q^{2k})^{i_{2k}}}. \tag{1}$$

By [27] i_k is the rank of homotopy group $\pi_k(G(A))$. The rational cohomology ring $H^*(G(A))$(even the rational homotopy type) is determined by the sequence $i_1, i_2, \cdots, i_k, \cdots$.

Set $\epsilon(A)=1$ or 0 depending on A is symmetrizable or not as in Kac[17]. By the results in [29][30], we have

Theorem 1: The Poincaré series of $F(A)$ is

$$P_{F(A)}(q)=\frac{\prod\limits_{k=1}^{\infty}(1-q^{2k})^{i_{2k-1}}}{(1-q^2)^n}\frac{1}{\prod\limits_{k=1}^{\infty}(1-q^{2k})^{i_{2k}}}. \tag{2}$$

Theorem 2: The sequence $i_1, i_2, \cdots, i_k, \cdots$ can be computed from $P_{F(A)}(q)$ and $\epsilon(A)$. In particular $i_1=i_2=0, i_3=\epsilon(A)$ and $i_{2k-1}=0$ for $k\geq 3$.

For the computation of i_{2k}, see [11][29][30].

In this paper we will give a combinatorics realization of the rank i_{2k} of homotopy group $\pi_k(G(A))$ for a large class of Kac-Moody groups. Since the computation for finite and affine cases has been obtained and for a decomposable Cartan matrix $A=A_1\oplus A_2$, $G(A)\cong G(A_1)\times G(A_2)$, we only consider the indecomposable and indefinite case.

The content of this paper is arranged as follows. In section 2, we give some results about the Hilbert series of graded associative algebras which will be used in the later sections. In section 3 we construct a Lie algebra $L(A)$ with grade for a Kac-Moody group $G(A)$ with certain good property. And we interpret i_{2k} as the dimension of the degree $2k$ component of $L(A)$. Since the realization is based on the Poincaré series of $F(A)$ we discuss the computation of Poincaré series of $F(A)$ in section 4. We are particularly interested in the case when the rank of Cartan matrix is 3 or 4. In section 5 we give some examples to show how our interpretation is implemented. In the last section we make a conjecture about the structure of grade Lie algebra $\pi_*(G(A))$ for certain type of Cartan matrix A.

2 Hilbert series of graded associative algebras

In this section we give some algebraic preparation. Our main reference for this section is Anick[1].

2.1 Hilbert series of free product

Let A be a graded associative algebra over a field K, then $A=\sum\limits_{i=0}^{\infty}A_k$, where A_k is the homogeneous degree k component of A. Denote the augmented ideal $\sum\limits_{i=1}^{\infty}A_k$ of A by $\widetilde{A}$. For two graded associative algebras A_1, A_2, denote by $A_1 * A_2$ the free product of A_1 and A_2. In this paper we consider only connected associative algebra A. That is $A_0\cong K$. The Hilbert series of A is $H_A(q)=\sum\limits_{i=0}^{\infty}q^k\dim A_k$. The Hilbert series satisfy the following property.

Lemma 1(Lemaire[22])Let A_1, A_2 be two connected graded associative algebras with Hilbert series H_{A_1}, H_{A_2}, then the Hilbert series of the free product $A = A_1 * A_2$ satisfies $\dfrac{1}{H_A} = \dfrac{1}{H_{A_1}} + \dfrac{1}{H_{A_2}} - 1$.

Proof: Let S_1, S_2 be the set of additive basis of augmented ideal $\widetilde{A}_1, \widetilde{A}_2$ respectively. Since A is the free product of A_1, A_2, we can construct a canonical basis S of $\widetilde{A}$ from S_1, S_2 whose elements are of the form of finite product $\alpha_{i_1}\alpha_{i_2}\cdots\alpha_{i_k}, k > 0$ such that if $\alpha_{i_j} \in S_1$ then $\alpha_{i_{j+1}} \in S_2$ and if $\alpha_{i_j} \in S_2$ then $\alpha_{i_{j+1}} \in S_1$ for $1 \leq j \leq k-1$. Let F be the subspace of $\widetilde{A}$ spanned by those elements of S starting from $\alpha_{i_1} \in S_1$ and G be the subspace of $\widetilde{A}$ spanned by those elements of S starting from $a_{i_1} \in S_2$. Then $A \cong K \oplus F \oplus G$.

Considering the Hilbert series of the two sides, we get

$$H_A = 1 + H_F + H_G.$$

It is obvious that $F = \widetilde{A}_1 A$ and $G = \widetilde{A}_2 A$. Hence $A = K \oplus \widetilde{A}_1 A \oplus \widetilde{A}_2 A$. We called this formula the first order expansion of A. It is easy to check that we have the following second order expansion of A

$$A = K \oplus \widetilde{A}_1 \oplus \widetilde{A}_2 \oplus \widetilde{A}_1\widetilde{A}_2 \oplus \widetilde{A}_2\widetilde{A}_1 \oplus \widetilde{A}_1\widetilde{A}_2 F \oplus \widetilde{A}_2\widetilde{A}_1 G.$$

So

$$H_A = 1 + H_{A_1} - 1 + H_{A_2} - 1 + 2(H_{A_1} - 1)(H_{A_2} - 1) + (H_{A_1} - 1)(H_{A_2} - 1)(H_F + H_G - 1)$$

Simplifying this formula, we get

$$\frac{1}{H_A} = \frac{1}{H_{A_1}} + \frac{1}{H_{A_2}} - 1.$$

Let $T(x_1, \cdots, x_m)$ be the tensor algebra generated by $x_1, \cdots, x_m$. Since $T(x_1, \cdots, x_m) \cong T(x_1) * \cdots * T(x_m)$ and for x with $\deg x = d, H_{T(x)} = \dfrac{1}{1 - q^d}$. We have

Corollary 1: For tensor algebra $A = T(x_1, \cdots, x_m)$ with $\deg x_i = d_i, 1 \leq i \leq m$, then

$$H_A = \frac{1}{1 - q^{d_1} - \cdots - q^{d_m}}.$$

2.2 Strongly free set

Let A be a graded associative algebra and B be a subalgebra of A, then the quotient homomorphism $\pi : A \to A/ABA$ is surjective. Let $\rho : A/ABA \to A$ be a chosen linear section of π, then there is a homomorphism $\mathrm{id} * \rho : B * (A/ABA) \to A$.

The following definition of strongly free set can be regarded as the generalization of the concept of regular sequences for commutative algebras to non-commutative algebras.

Definition 1(Anick[1])Let A be a graded associative algebra and B be a subalgebra of A, B is called a weak summand of A if the homomorphism $\mathrm{id} * \rho : B * (A/ABA) \to A$ is an isomorphism of K-vector spaces. Let $\alpha = \{\alpha_1, \cdots, \alpha_k\}$ be a graded set in A, α is called a strongly free set in A if the subalgebra $K\langle\alpha\rangle$ generated by α in A is a free algebra and $K\langle\alpha\rangle$ is a weak summand of A.

Let α be a strongly free set in A, α generates an ideal $A\alpha A \subset A$ in A, the following lemma gives the relation between the Hilbert series H_A and $H_{A/A\alpha A}$.

Lemma 2: Let A be a connected graded associative algebra and $\alpha = \{\alpha_1, \cdots, \alpha_n\}$ be a strongly free set in A. If the degrees of elements in α are $e_1, \cdots, e_n$, then $\frac{1}{H_{A/A\alpha A}} = \frac{1}{H_A} + q^{e_1} + \cdots + q^{e_n}$.

Proof: Since $\mathrm{id} * \rho : T(\alpha) * (A/ABA) \to A$ is an isomorphism of vector spaces, we get

$$\frac{1}{H_A} = \frac{1}{H_{A/A\alpha A}} + \frac{1}{H_{K\langle\alpha\rangle}} - 1$$

Combing with $H_{K\langle\alpha\rangle} = \frac{1}{1 - e^{e_1} - \cdots - q^{e_l}}$, we prove the lemma.

For a connected graded associative algebra A generated by set $X = \{x_1, \cdots, x_m\}$, then

$$A \cong T(x_1, \cdots, x_m)/I$$

where I is the ideal of relation of A with respect to generators set X. If I is generated by a strongly free set in $T(x_1, \cdots, x_m)$, then we have the following result.

Corollary 2: Let A be a connected graded associative algebra with generator set $X = \{x_1, \cdots, x_m\}$ and relation set $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_n\}$. If α is a strongly free set, then the Hilbert series of A is

$$H_A = \frac{1}{1 - q^{d_1} - \cdots - q^{d_m} + q^{e_1} + \cdots + q^{e_n}}.$$

where $d_1, d_2 \cdots, d_m$ are the degrees of set $\{x_1, \cdots, x_m\}$ and $e_1, e_2 \cdots, e_n$ are the degrees of set $\{\alpha_1, \alpha_2, \cdots, \alpha_n\}$.

2.3 Hilbert series of universal enveloping algebra of Lie algebra with grade

Let $L = \bigoplus_{i=1}^{\infty} L_i$ be a Lie algebra and $[L_k, L_l] \subset L_{k+l}$ for all $k, l > 0$, then we say L is a Lie algebra with grade. For a Lie algebra L with grade, its universal enveloping algebra $U(L)$ is an graded associative Hopf algebra. The coproduct on $U(L)$ is defined by $\delta(x) = 1 \otimes x + x \otimes 1$ for $x \in L$ and δ is cocommutative. Therefore By [23] $U(L)$ is primitively generated. For a free Lie algebra L with grade generated by $X = \{x_1, x_2, \cdots, x_m\}$, the universal enveloping algebra is the tensor algebra $T(x_1, x_2, \cdots, x_m)$.

Lemma 3: Let $L = \bigoplus_{k=1}^{\infty} L_k$ be a Lie algebra with grade and $j_k = \dim L_k$, then the Hilbert series $H_{U(L)} = \frac{1}{\prod_{k=1}^{\infty} (1 - q^k)^{j_k}}$.

This lemma is derived directly from the Poincaré-Birkhoff-Witt Theorem.

Let L be a Lie algebra with grade and $\alpha \subset A$ be a graded set and J be the quotient Lie algebra of L with respect to the ideal I generated by α, then we have

Lemma 4: The universal enveloping algebra $U(J)$ is isomorphic to the quotient Hopf algebra of $U(L)$ with respect to the ideal $U(L)IU(L)$.

For the proof of this lemma, see Bourbaki [7].

Definition 2: Let L be a Lie algebra with grade, $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_m\} \subset L$ is called a strongly free set in L if and only if the image of α in $U(L)$ is a strongly free set.

Lemma 5: Let L be a Lie algebra with grade generated by graded set $X = \{x_1, \cdots, x_m\}$ with defining relation set $\alpha = \{\alpha_1, \cdots, \alpha_n\}$. If α is strongly free set, then the Hilbert series of $U(L)$ is

$$\frac{1}{1 - q^{d_1} - \cdots - q^{d_m} + q^{e_1} + \cdots + q^{e_n}} = \frac{1}{\prod\limits_{k=1}^{\infty} (1 - q^k)^{j_k}}.$$

where $d_1, d_2 \cdots, d_m$ are the degrees of set $\{x_1, \cdots, x_m\}$ and $e_1, e_2 \cdots, e_n$ are the degrees of set $\{\alpha_1, \alpha_2, \cdots, \alpha_n\}$.

3 Rank of homotopy group $\pi_k(G(A))$

3.1 The theorem of Milnor and Moore

In this section we use the results in previous section to discuss the rank i_k of homotopy group $\pi_k(G(A))$.

For a connected Hopf space G with unit and homotopy associative multiplication, the homology $H_*(G)$ is a Hopf algebra. The diagonal $\Delta : G \to G \times G$ is co-commutative, implying that the coproduct of $H_*(G)$ is commutative. Hence by [23] $H_*(G)$ is primitively generated.

On the rational homotopy group $\pi_*(G)$, the Samelson product $[\ ,\ ] : \pi_p(G) \times \pi_q(G) \to \pi_{p+q}(G)$ is defined as

$$[\alpha, \beta](s \wedge t) = \alpha(s)\beta(t)\alpha(s)^{-1}\beta(t)^{-1}, s \in S^p, t \in S^q.$$

$\pi_*(G)$ forms a graded Lie algebra over $\mathbb{Q}$ with Samelson product.

Theorem(Milnor-Moore) Let G be a connected homotopy associative H-space with unit and $\chi : \pi_*(G) \to H_*(G)$ be the Hurewicz morphism of graded Lie algebras, then the induced morphism $\widetilde{\chi} : U(\pi_*(G)) \to H_*(G)$ is an isomorphism of Hopf algebras. Where $U(\pi_*(G)))$ is the enveloping algebra of $\pi_*(G)$.

By this theorem, to determine the rank i_k of $\pi_k(G(A))$, we only need to consider the Hilbert series of $H_*(G(A))$. Since the Hilbert series of $H_*(G(A))$ contains the same information as the Poincaré series of $G(A)$, we discuss the Poincaré series of $G(A)$ for convenience. $H_*(G(A))$ is the tensor product of a polynomial algebra with even degree generators and an exterior algebra with odd degree generators. In this paper we only consider indecomposable and indefinite Cartan matrix A. In this case it is proved in [29] that if A is symmetrizable then the exterior algebra part of $H^*(G(A))$ is generated by one degree 3 generators and if A is not symmetrizable then $H^*(G(A))$ has no exterior algebra part.

3.2 Chow ring of $G(A)$

Lie algebra $\pi_*(G(A))$ is a graded Lie algebra whose universal enveloping algebra is $H_*(G(A))$. For the Lie sub-algebra $\pi_{even}(G(A)) = \sum\limits_{i=1}^{\infty} \pi_{2k}(G(A))$, the universal enveloping algebra is $H_{even}(G(A))$. The dual Hopf algebra of $H^{even}(G(A)$ is isomorphism to the Chow ring $\mathrm{Ch}^*(G(A))$. As algebra $\mathrm{Ch}^*(G(A))$ is the subalgebra of $H^*(G(A))$ generated by even dimensional generators of degree great than 2. By relating $\pi_{even}(G(A))$ with Chow ring of $G(A)$ we transform the computation of i_{2k} to the computation of Hilbert series of Chow ring.

Lemma 6: The Hilbert series of $\mathrm{Ch}^*(G(A))$ is

$$C_A(q) = P_{F(A)}(q)(1-q^2)^n(1-q^4)^{-\epsilon(A)}$$

and

$$C_A(q) = \prod_{k=2}^{\infty} \frac{1}{(1-q^{2k})^{i_{2k}}} \tag{3}$$

For reference see Kac[17].

3.3 Realization of i_k

The Poincaré series $P_{F(A)}(q)$ is of the form $\dfrac{\prod_{i=1}^{r}[t_i]}{Q(q^2)}$, where $Q(q)$ is a polynomial of q with constant item 1 and $[d_i] = \dfrac{1-q^{2t_i}}{1-q^2}$. By [11] $\prod_{i=1}^{r}[t_i]$ is in fact the least common multiple of those Poincaré series of flag manifolds associated to the finite type principal sub-matrices of A. We assume the polynomial $Q(q)$ is of the form $1-a_1q^{d_1}-\cdots-a_mq^{d_m}+b_1q^{e_1}+\cdots+b_nq^{e_n}$ with $a_i>0, 1\le i\le m; b_j>0, 1\le j\le n$ and $d_1<d_2<\cdots<d_m, e_1<e_2<\cdots<e_n$.

We have

$$C_A(q) = \prod_{k=2}^{\infty} \frac{1}{(1-q^{2k})^{i_{2k}}}.$$

and by Zhao-Jin[30] there exists a unique sequence $j_1, j_2, \cdots, j_k \cdots$ such that

$$\frac{1}{Q(q)} = \prod_{k=1}^{\infty} \frac{1}{(1-q^k)^{j_k}}.$$

Substituting the above two formulas into

$$C_A(q) = \frac{(1-q^2)^{n-r}(1-q^4)^{-\epsilon(A)}\prod_{i=1}^{r}(1-q^{2t_i})}{Q(q^2)}. \tag{4}$$

We get:

Lemma 7: The series i_k satisfy $i_2=0, i_4=j_2-l_2+\epsilon(A), i_{2k}=j_k-l_k, k>2$, where $l_k=\#\{i|t_i=k, 1\le i\le r\}$.

We give the following definition.

Definition 3: A polynomial $Q(q)$ is called a strongly positive polynomial if there exists a free Lie algebra L with grade and a strongly free set $\alpha\in L$, such that $\dfrac{1}{Q(q)}$ is the Hilbert series of the quotient algebra of the universal enveloping algebra $U(L)$ with respect to the ideal generated by $\alpha\in L$. A Kac-Moody groups is called a good Kac-Moody group if it correspond to a strongly positive polynomials $Q(q)$

A large class of Kac-Moody groups are good. For a good Kac-Moody groups $G(A)$, there exists a Lie algebra $L(A)$ such that the Hilbert series of $U(L)$ is $Q(q)$. In this case j_k is just the dimension of degree k homogeneous component of $L(A)$. So we have

Theorem 3: For a good Kac-Moody groups $G(A)$, the rank i_k of the homotopy group $\pi_k(G(A))$ satisfy $i_2 = 0, i_4 = j_2 - l_2 + \epsilon(A), i_{2k} = j_k - l_k, k > 2$, where j_k is the rank of the degree k component of Lie algebra $L(A)$ and $l_k = \#\{i | t_i = k, 1 \le i \le r\}$.

4 The computation of Poincaré series $P_{F(A)}(q)$

We need an algorithm to compute the Poincaré series of flag manifolds.

4.1 General results about the Poincaré series of flag manifolds

The Weyl group $W(A)$ associated to a Cartan matrix A is the group generated by the Weyl reflections $\sigma_i : h^* \to h^*$ with respect to simple co-roots $\alpha_i^\vee, 1 \le i \le n$, where $\sigma_i(\alpha) = \alpha - \langle \alpha, \alpha_i^\vee \rangle \alpha_i$. $W(A)$ has a Coxeter presentation

$$W(A) =< \sigma_1, \cdots, \sigma_n | \sigma_i^2 = e, 1 \le i \le n; (\sigma_i\sigma_j)^{m_{ij}} = e, 1 \le i < j \le n > .$$

where $m_{ij} = 2, 3, 4, 6$ or ∞ as $a_{ij}a_{ji} = 0, 1, 2, 3$ or ≥ 4 respectively. For details see Kac[13], Humphreys[9].

Each element $w \in W(A)$ has a decomposition of the form $w = \sigma_{i_1} \cdots \sigma_{i_k}, 1 \le i_1, \cdots, i_k \le n$. The length of w is defined as the least integer k in all of those decompositions of w, denoted by $l(w)$. The Poincaré series of $g(A)$ is the power series $P_A(q) = \sum\limits_{w \in W(A)} q^{2l(w)}$.

Steinberg[26] proved that the Poincaré series $P_A(q)$ of the flag manifold $F(A)$ of a Lie group $G(A)$ is a rational function. And the result is easy to extend to the Poincaré series of a general Kac-Moody group $G(A)$ or its flag manifolds $F(A)$.

In [11] the authors discussed the computation of Poincaré series of flag manifolds of Kac-Moody flag groups. Let A be an $n \times n$ Cartan matrix, $S = \{1, 2, \cdots, n\}$. For each $I \subset S$, let A_I be the Cartan matrix $(a_{ij})_{i,j \in I}$. Let $\dim A_I$ be the complex dimension of the flag manifolds $F(A_I)$. The following lemma is used in this paper.

Lemma 8(Steinberg[26])Let A be an indefinite Cartan matrix, then we have

$$\sum_{I \subset S} (-1)^{|I|} \frac{P_{F(A)}(q)}{P_{F(A_I)}(q)} = 0 \tag{5}$$

and

$$P_{F(A)}(q^{-1}) = \sum_{I \subset S, \dim A_I < \infty} \frac{(-1)^{|I|}}{P_{F(A_I)}(q)} \tag{6}$$

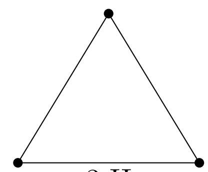
The Poincaré series $P_{F(A)}(q)$ can be computed through Steinberg's formula by a recurrence procedure.

4.2 Poincaré series of flag manifolds of rank 3 and 4

By the results in [10][11], for a Cartan matrix A the Poincaré series $P_{F(A)}(q)$ is determined by the Coxeter graph $\Gamma(A)$. For a Coxeter graph $\Gamma(A)$ we define the reduced graph is the graph obtained by replacing all the k-fold edges between pairs of vertices by one-fold edge.

Lemma 9: Let A be an indecomposable rank 3 Cartan matrix, then its reduced Coxeter graph is of the following two types.

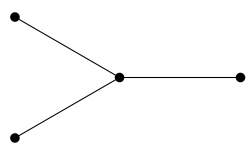
3-I



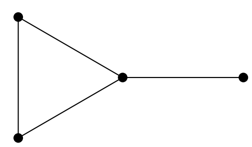
3-II

Lemma 10: Let A be an indecomposable rank 4 Cartan matrix, then its Coxeter graph is of the following five types.

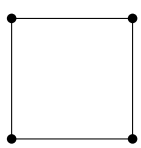
4-I



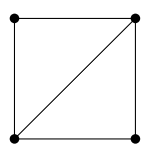
4-II



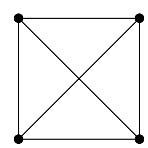
4-III



4-IV



4-V



4-VI

The automorphisms of these graphs are: 3-I, $\mathbb{Z}_2$; 3-II, S_3; 4-I, $\mathbb{Z}_2$; 4-II, S_3; 4-III, $\mathbb{Z}_2$; 4-IV, D_4; 4-V, $\mathbb{Z}_2 \times \mathbb{Z}_2$; 4-VI, S_4.

The computation results for the Poincaré series of $F(A)$ for rank 3 and 4 Cartan matrices are listed in the appendix A.

5 Some examples

Example 1: For Cartan matrix $A = \begin{pmatrix} 2 & -1 & -1 \\ -3 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$, $\epsilon(A) = 0$. The Poincaré series of $F(A)$ is

$$P_{F(A)}(q) = \frac{[2][6]}{t^{12} - t^{10} - t^8 + t^6 - t^4 - t^2 + 1}.$$

In this example $Q(q) = t^6 - t^5 - t^4 + t^3 - t^2 - t + 1$, we define a Lie algebra $L(A) = \langle x_1, x_2, x_4, x_5 | [x_1, x_2], [x_1, x_5] \rangle$ with degree i generator x_i for $i = 1, 2, 4, 5$, then $[x_1, x_2], [x_1, x_5]$ form

a strongly free set in L. Let the dimension of degree k component of $L(A)$ be j_k, then by Theorem 3 we have $i_2 = 0, i_4 = j_2 - 1, i_{12} = j_6 - 1$ and $i_{2k} = j_k$ for $k \neq 1, 2, 6$.

Example 2: For Cartan matrix $A = \begin{pmatrix} 2 & -1 & -1 & -1 \\ -2 & 2 & -1 & -1 \\ -1 & -1 & 2 & -1 \\ -1 & -1 & -1 & 2 \end{pmatrix}$, the $\epsilon(A) = 0$. The Poincaré series of $F(A)$ is

$$P_{F(A)}(q) = \frac{[2][3][4]}{3t^{12} + t^{10} - t^8 - t^6 - 3t^4 - t^2 + 1}.$$

So $Q(q) = 3t^6 + t^5 - t^4 - t^3 - 3t^2 - t + 1$, we define a Lie algebra

$L(A) = \langle x_1, x_{21}, x_{22}, x_{23}, x_4 | [x_1, x_4], [x_{21}, x_4], [x_{22}, x_4], [x_{23}, x_4] \rangle$.

Where $\deg x_1 = 1, \deg x_4 = 4, \deg x_{2i} = 2, \forall i$. Then $[x_1, x_4], [x_{21}, x_4], [x_{22}, x_4], [x_{23}, x_4]$ form a strongly free set in L and we have $i_2 = 0, i_4 = j_2 - 1, i_6 = j_3 - 1, i_8 = j_4 - 1$ and $i_{2k} = j_k$ for $k \neq 1, 2, 3, 4$.

6 Criteria for strongly free set

There is no easily applied criterion to determine whether or not a given graded set α in an algebra A is strongly free. But for free algebras Anick gave some criteria in [1]. We cite the corresponding results.

Definition 4: Let S be any locally finite graded set and let B be the free monoid on S. A set of monomials $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_n\} \subset B - \{1\}$ is combinatorially free iff (a) no α_i is a sub-monomial of α_j for $i \neq j$ and (b) whenever $\alpha_i = x_1 y_1$, and $\alpha_j = x_2 y_2$ for $x_1, y_1, x_2, y_2 \in B - \{1\}$ we have $y_1 \neq x_2$.

Condition (b) says that the beginning of one monomial cannot be the same as the ending of another (or the same) monomial.

Theorem 4: (Anick[1]) Let $A = K(S)$, let B be the free monoid on S and suppose $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_n\} \subset B - \{1\}$ is a set of monomials. Then α is strongly free in A if and only if α is combinatorially free.

In the following, we give a monomials basis M for a free monoid $B = K(S)$. If S is empty, then $B = K$ and the only monomial is 1. Otherwise choose any total ordering on S. Define an ordering e on B as follows. For monomial$x, y \in B, x < y$ iff $e(x) < e(y)$; if $e(x) = e(y)$, compare x and y using the lexicographic ordering induced on B by the ordering for S. Since (S, e) is locally finite, $e^{-1}(n) \cap B$ is finite for each n, and B is isomorphic as an ordered set to the positive integers. This ordering has the additional property that if $u, w, x, y \in B$ and $x < y$, then $uxw < vyw$.

Given a nonzero element $x \in K(S)$, write x as a linear combination of monomials $x = c_1 y_1 + \cdots + c_l y_l$, where $c_i \in K$. If y_i is the largest monomial(in the sense of the ordering of the monomials) for which $c_i \neq 0$, then y_i is called the high term of x.

Theorem 5: (Anick[1])Let $A = K(S)$ and suppose $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_n\} \subset \widetilde{A} - \{0\}$. Using any fixed ordering on S, let $\hat{\alpha}_i$ be the high term of α_i for each $\alpha_i \in \alpha$, then α is strongly free in A if $\hat{\alpha} = \{\hat{\alpha}_1, \hat{\alpha}_2, \cdots, \hat{\alpha}_n\}$ is combinatorially free.

7 A conjecture on the structure of $\pi_*(G(A))$

Example 3: Let A be a rank n Cartan matrix A which satisfy $a_{ij}a_{ji} \geq 4$ for all $i \neq j$, then the Weyl group of $G(A)$ is

$$W(A) = < \sigma_1, \cdots, \sigma_n | \sigma_i^2 = 1, 1 \leq i \leq n > .$$

By Lemma 2, $\frac{1}{H_A} = \frac{n}{1+q} - (n-1)$, so $H_A = \frac{1+q}{1-(n-1)q}$.

$$C_A(q) = \frac{(1-q)^{n-1}}{1-(n-1)q} = \frac{1}{\frac{1-(n-1)q}{(1-q)^{n-1}}} = \frac{1}{1 - a_2q^2 - a_3q^3 - \cdots - a_kq^k - \cdots}.$$

where $a_k = (n-1)\binom{k+n-3}{k-1} - \binom{k+n-2}{k}$.

Lemma 11: $a_k > 0$ for all $k > 1$.

Conjecture : $\pi_{even}(G(A))$ is a free Lie algebra with a_k free generators of degree $2k$ for each $k > 1$.

References

[1] D. J. Anick, Noncommutative graded algebras and their Hilbert series. J. Algebra 78 (1982), no. 1, 120-140.

[2] A. Borel, Sur la cohomologie des espaces fibres principaux et des espaces homogenes de groupes de Lie compacts, Ann.math. 57(1953), 115-207.

[3] A. Borel, Homology and cohomology of compact connected Lie groups. Proc. Nat. Acad. Sci. U. S. A. 39(1953). 1142-1146.

[4] A. Borel, Sur lhomologie et la cohomologie des groupes de Lie compacts connexes, Amer. J. Math. 76(1954), 273-342.

[5] R. Bott, H. Samelson, The cohomology ring of G/T. Proc. Nat. Acad. Sci. U. S. A. 41 (1955), 490-493.

[6] R. Bott, An Application of the Morse Theory to the topology of Lie-groups, Bull. Soc. Math. France 84, 1956, 251-281

[7] N. Bourbaki, Lie groups and Lie algebras. Chapters 1C3. Translated from the French. Reprint of the 1989 English translation. Elements of Mathematics (Berlin). Springer-Verlag Berlin, 1998.

[8] H. Hopf, Uber die topologie der gruppenmannigfaltigkeiten und ibre veraligemeinerungen, Ann. of Math. 42(1941), 22-52.

[9] J. E. Humphreys, Reflection Groups and Coxeter Groups, Cambridge University Press, 1990

[10] Jin chunhua, Poincaré series of Kac-Moody Lie algebra, Master thesis of Beijing normal university, 2010.

[11] Jin chunhua, Zhao Xu-an, On the Poincaré series of Kac-Moody Lie algebra, arXiv: 1210.0648v1.

[12] V. Kac, Simple irreducible graded Lie algebras of finite growth. (Russian) Izv. Akad. Nauk SSSR Ser. Mat. 32 1968 1323-1367.

[13] V. Kac, Infinite Dimensional Lie Algebras, Cambridge University Press, 1982

[14] V. Kac, D. Peterson, Regular functions on certain infinite-dimensional groups. Arithmetic and geometry, Vol. II, 141-166, Progr. Math., 36, Birkhäuser Boston, Boston, MA, 1983.

[15] V. Kac; D. Peterson, Defining relations of certain infinite-dimensional groups. The mathematical heritage of lie Cartan (Lyon, 1984). Astrisque 1985, Numro Hors Srie, 165-208.

[16] V. Kac, Constructing groups associated to infinite-dimensional Lie algebras. Infinite dimensional groups with applications (Berkeley, Calif., 1984), 167-216, Math. Sci. Res. Inst. Publ., 4, Springer, New York, 1985.

[17] V. Kac, Torsion in cohomology of compact Lie groups and Chow rings of reductive algebraic groups. Invent. Math. 80 (1985), no. 1, 69-79.

[18] N. R. Kitchloo, Topology of Kac-Moody groups, Ph D thesis of California Institute of Technology, 1998.

[19] B. Kostant, S. Kumar, The nil Hecke ring and cohomology of G/P for a Kac-Moody group G, Adv. in Math. 62 (1986), no. 3, 187-237.

[20] S. Kumar, Rational homotopy theory of flag varieties associated to Kac-Moody groups. Infinite-dimensional groups with applications (Berkeley, Calif., 1984), 233-273, Math. Sci. Res. Inst. Publ., 4, Springer, New York, 1985.

[21] S. Kumar, Kac-Moody groups, their flag varieties and representation theory. (English summary) Progress in Mathematics, 204. Birkhäauser Boston, Inc., Boston, MA, 2002.

[22] J. M. Lemaire, Algèbres connexes et homologie des espaces de lacets. (French) Lecture Notes in Mathematics, Vol. 422. Springer-Verlag, Berlin-New York, 1974.

[23] J. W. Milnor, J. C. Moore, On the structure of Hopf algebras. Ann. of Math. (2) 81 1965 211-264.

[24] R. V. Moody, A new class of Lie algebras. J. Algebra 10 1968 211-230.

[25] L. S. Pontryagin, Sur les nombres de Betti des groupes de Lie, C. R. Acad. Sci. Paris, 200(1935), 1277-1280.

[26] R. Steinberg, Endomorphisms of linear algebraic groups, Memoirs Amer. Math. Soc. 80 (1968) 1-108; also available in Collected papers. Amer. Math. Soc. (1997).

[27] D. Sullivan, Infinitesimal computations in topology. Inst. Hautes tudes Sci. Publ. Math. No. 47 (1977), 269-331 (1978).

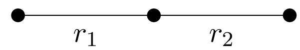
[28] G. W. Whitehead, Elements of homotopy theory, Springer-Verlag,1978.

[29] Zhao Xu-an, Jin Chunhua, Polynomial invariants of Weyl groups for Kac-Moody groups, arXiv:1211.3197.

[30] Zhao Xu-an, Jin Chunhua, Poincaré series and rational cohomology rings of Kac-Moody groups and their flag manifolds, arXiv:1301.2647.

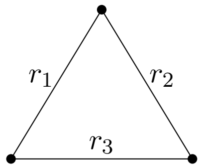
8 Appendix-the lists of Poincaré series of rank 3 and 4

The Poincaré series of Cartan matrices with reduced Coxeter graph 3-I



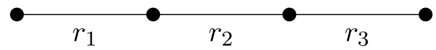
	$[r1, r2]$	*Poincaré series*
1	$[1,1]$	$\frac{\left(t^2-1\right)\left(t^3-1\right)\left(t^4-1\right)}{(t-1)^3}$
2	$[1,2]$	$\frac{\left(t^2-1\right)\left(t^4-1\right)\left(t^6-1\right)}{(t-1)^3}$
3	$[1,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-t^5-t+1}$
4	$[1,4]$	$-\frac{\left(t^2+t+1\right)(t+1)^2}{t^3+t^2-1}$
5	$[2,2]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{t^4-t^3-t+1}$
6	$[2,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-t^7+t^6-2\,t^5+t^4-2\,t^3+t^2-t+1}$
7	$[2,4]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)}{t^3+t-1}$
8	$[3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-t^5-t^3-t+1}$
9	$[3,4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^5+t^3+t-1}$
10	$[4,4]$	$-\frac{(t+1)^2}{t^2+t-1}$

The Poincaré series of Cartan matrices with reduced Coxeter graph 3-II



	$[r1, r2, r3]$	$Poincaré\,series$
11	$[1,1,1]$	$\frac{t^2+t+1}{t^2-2\,t+1}$
12	$[1,1,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^4-t^3-t^2+1}$
13	$[1,1,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-t^5-t^4+t^3-t^2-t+1}$
14	$[1,1,4]$	$-\frac{\left(t^2+t+1\right)(t+1)}{t^2+t-1}$
15	$[1,2,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^4-2\,t^3-t^2+1}$
16	$[1,2,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-t^7-t^5-t^4-t^3-t+1}$
17	$[1,2,4]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^5+2\,t^4+2\,t^3+t^2-1}$
18	$[1,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-t^5-t^4-t^2-t+1}$
19	$[1,3,4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^5+t^4+t^2+t-1}$
20	$[1,4,4]$	$-\frac{\left(t^2+t+1\right)(t+1)}{t^3+t^2+t-1}$
21	$[2,2,2]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{t^4-t^3-t^2-t+1}$
22	$[2,2,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-t^7-2\,t^5-2\,t^3-t+1}$
23	$[2,2,4]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)}{t^3+t^2+t-1}$
24	$[2,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-t^7-2\,t^5-t^4-2\,t^3-t+1}$
25	$[2,3,4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^7+t^6+2\,t^5+t^4+2\,t^3+t-1}$
26	$[2,4,4]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)}{t^4+t^3+t^2+t-1}$
27	$[3,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-t^5-t^4-t^3-t^2-t+1}$
28	$[3,3,4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^5+t^4+t^3+t^2+t-1}$
29	$[3,4,4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6+t^5+t^4+t^3+t^2+t-1}$
30	$[4,4,4]$	$-\frac{t+1}{2\,t-1}$

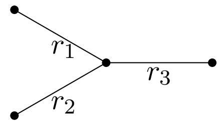
The Poincaré series of Cartan matrices with reduced Coxeter graph 4-I



	$[r1, r2, r3]$	*Poincaré series*
1	$[1,1,1]$	$\frac{(t^2-1)(t^3-1)(t^4-1)(t^5-1)}{(t-1)^4}$
2	$[1,1,2]$	$\frac{(t^2-1)(t^4-1)(t^6-1)(t^8-1)}{(t-1)^4}$
3	$[1,1,3]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)(t+1)}{t^8-t^4-t+1}$
4	$[1,1,4]$	$-\frac{(t^2+t+1)(t^3+t^2+t+1)(t+1)}{t^3+t-1}$
5	$[1,2,1]$	$\frac{(t^2-1)(t^6-1)(t^8-1)(t^{12}-1)}{(t-1)^4}$
6	$[1,2,2]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)(t+1)}{t^8-t^3-t+1}$
7	$[1,2,3]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)(t+1)}{t^8-t^4-t^3-t+1}$
8	$[1,2,4]$	$-\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)(t+1)}{t^6+t^4+t^3+t-1}$
9	$[1,3,1]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)}{t^6-t^4+t^2-2t+1}$
10	$[1,3,2]$	$\frac{(t^3+t^2+t+1)(t^5+t^4+t^3+t^2+t+1)(t+1)}{t^9+t^8-t^5-t^4-t^3-t+1}$
11	$[1,3,3]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)^2}{t^7+t^6-t^5-t^4-t^2-t+1}$
12	$[1,3,4]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)}{t^6-t^5-t^3+t^2-2t+1}$
13	$[1,4,1]$	$-\frac{(t^2+t+1)(t+1)^2}{t^2+t-1}$
14	$[1,4,2]$	$-\frac{(t^3+t^2+t+1)(t^2+t+1)(t+1)^2}{t^5+2t^4+2t^3+t^2-1}$
15	$[1,4,3]$	$-\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)^2}{t^5+t^4+t^2+t-1}$
16	$[1,4,4]$	$-\frac{(t^2+t+1)(t+1)^2}{t^3+t^2+t-1}$
17	$[2,1,2]$	$-\frac{(t^3+1)(t^3+t^2+t+1)(t+1)}{t^7-2t^6+t^5-t^2+2t-1}$
18	$[2,1,3]$	$\frac{(t^3+t^2+t+1)(t^5+t^4+t^3+t^2+t+1)(t+1)}{t^8+t^6-t^5-t^3-t+1}$
19	$[2,1,4]$	$-\frac{(t^2+t+1)(t^3+1)(t^3+t^2+t+1)(t+1)}{t^5+t^3+t-1}$
20	$[2,2,2]$	$\frac{(t^3+t^2+t+1)(t+1)^2}{t^5+t^4-t^3-t^2-t+1}$

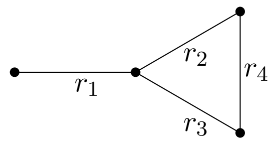
	$[r1, r2, r3]$	*Poincaré series*
21	$[2,2,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9+t^8-t^5-2\,t^3-t+1}$
22	$[2,2,4]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)^2}{t^5-t^3-t^2-t+1}$
23	$[2,3,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9+t^8-2\,t^5-2\,t^3-t+1}$
24	$[2,3,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9+t^8-2\,t^5-t^4-2\,t^3-t+1}$
25	$[2,3,4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9-t^6-2\,t^5-t^4-2\,t^3-t+1}$
26	$[2,4,2]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)^2}{t^3+t^2+t-1}$
27	$[2,4,3]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^7+t^6+2\,t^5+t^4+2\,t^3+t-1}$
28	$[2,4,4]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)^2}{t^4+t^3+t^2+t-1}$
29	$[3,1,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{t^7+t^6-t^5-t^2-t+1}$
30	$[3,1,4]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)^2}{t^7-t^5-t^2-t+1}$
31	$[3,2,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9+t^8-t^5-t^4-2\,t^3-t+1}$
32	$[3,2,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9-t^6-t^5-t^4-2\,t^3-t+1}$
33	$[3,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{t^7+t^6-t^5-t^4-t^3-t^2-t+1}$
34	$[3,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{t^7-t^5-t^4-t^3-t^2-t+1}$
35	$[3,4,3]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{t^5+t^4+t^3+t^2+t-1}$
36	$[3,4,4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{t^6+t^5+t^4+t^3+t^2+t-1}$
37	$[4,1,4]$	$\frac{\left(t^2+t+1\right)(t+1)^2}{t^4-t^3-t^2-t+1}$
38	$[4,2,4]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{t^4-2\,t^3+t^2-2\,t+1}$
39	$[4,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5+t^4-2\,t^3+t^2-2\,t+1}$
40	$[4,4,4]$	$-\frac{(t+1)^2}{2\,t-1}$

The Poincaré series of Cartan matrices with reduced Coxeter graph 4-II



	$[r1, r2, r3]$	*Poincaré series*
41	$[1,1,1]$	$\frac{\left(t^2-1\right)\left(t^4-1\right)^2\left(t^6-1\right)}{(t-1)^4}$
42	$[1,1,2]$	$-\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^7-2\,t^6+t^5-t^2+2\,t-1}$
43	$[1,1,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9+t^8+t^6-2\,t^5-t^3-t+1}$
44	$[1,1,4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)^2}{t^5+t^4+2\,t^3+t^2-1}$
45	$[1,2,2]$	$\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^6-t^5+t^4-t^3+t^2-2\,t+1}$
46	$[1,2,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9+t^8+t^6-2\,t^5-2\,t^3-t+1}$
47	$[1,2,4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^7+2\,t^5+2\,t^3+t-1}$
48	$[1,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{2\,t^7+t^6-2\,t^5-t^3-t^2-t+1}$
49	$[1,3,4]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)^2}{t^7-2\,t^5-t^3-t^2-t+1}$
50	$[1,4,4]$	$-\frac{\left(t^2+t+1\right)(t+1)^3}{t^4+3\,t^3+2\,t^2-1}$
51	$[2,2,2]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)^2}{2\,t^5+t^4-2\,t^3-t^2-t+1}$
52	$[2,2,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-2\,t^5-3\,t^3-t+1}$
53	$[2,2,4]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)^2}{t^5-2\,t^3-t^2-t+1}$
54	$[2,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-3\,t^5-t^4-3\,t^3-t+1}$
55	$[2,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9-t^7-t^6-3\,t^5-t^4-3\,t^3-t+1}$
56	$[2,4,4]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)^2}{t^4+2\,t^3+t^2+t-1}$
57	$[3,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{2\,t^7+t^6-2\,t^5-t^4-2\,t^3-t^2-t+1}$
58	$[3,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{t^7-2\,t^5-t^4-2\,t^3-t^2-t+1}$
59	$[3,4,4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{t^6+2\,t^5+t^4+2\,t^3+t^2+t-1}$
60	$[4,4,4]$	$-\frac{(t+1)^3}{t^3+2\,t^2+t-1}$

The Poincaré series of Cartan matrices with reduced Coxeter graph 4-III



	$[r1, r2, r3, r4]$	*Poincaré series*
61	$[1,1,1,1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^4-t^3+t^2-2\,t+1}$
62	$[1,1,1,2]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)^2}{t^6+t^5-2\,t^3-2\,t^2+1}$
63	$[1,1,1,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^8-t^5-t^4-t^2-t+1}$
64	$[1,1,1,4]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^3+t^2+t-1}$
65	$[1,1,2,1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^8-t^4-t^2-t+1}$
66	$[1,1,2,2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^8-t^5-t^3-t^2-t+1}$
67	$[1,1,2,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^8-t^5-t^4-t^3-t^2-t+1}$
68	$[1,1,2,4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^6+t^5+t^4+t^3+t^2+t-1}$
69	$[1,1,3,1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-t^4-t^3+t^2-2\,t+1}$
70	$[1,1,3,2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9+t^8-2\,t^5-t^4-t^3-t^2-t+1}$
71	$[1,1,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9+t^8-2\,t^5-2\,t^4-t^3-t^2-t+1}$
72	$[1,1,3,4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{t^8-t^7+t^6-2\,t^5-2\,t^3+t^2-2\,t+1}$
73	$[1,1,4,1]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^4+t^3+t^2+t-1}]$
74	$[1,1,4,2]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)^2}{t^5+2\,t^4+3\,t^3+2\,t^2-1}$
75	$[1,1,4,3]$	$-\frac{\left(t^2+t+1\right)\left(t^3+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^7+t^6+2\,t^5+2\,t^4+t^3+t^2+t-1}$
76	$[1,1,4,4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^5+t^4+2\,t^3+t^2+t-1}$
77	$[1,2,2,1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^8-t^4-t^3-t^2-t+1}$
78	$[1,2,2,2]$	$\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^6-t^5-2\,t+1}$
79	$[1,2,2,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^8-t^5-t^4-2\,t^3-t^2-t+1}$
80	$[1,2,2,4]$	$-\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^4+2\,t-1}$

	$[r1,r2,r3,r4]$	*Poincaré series*
81	$[1,2,3,1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9+t^8-t^5-2\,t^4-t^3-t^2-t+1}$
82	$[1,2,3,2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9+t^8-2\,t^5-t^4-2\,t^3-t^2-t+1}$
83	$[1,2,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9+t^8-2\,t^5-2\,t^4-2\,t^3-t^2-t+1}$
84	$[1,2,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9-t^6-2\,t^5-2\,t^4-2\,t^3-t^2-t+1}$
85	$[1,2,4,1]$	$-\frac{\left(t^2+t+1\right)\left(t^3+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^7+t^6+t^5+2\,t^4+t^3+t^2+t-1}$
86	$[1,2,4,2]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^7+t^6+2\,t^5+t^4+2\,t^3+t^2+t-1}$
87	$[1,2,4,3]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^7+t^6+2\,t^5+2\,t^4+2\,t^3+t^2+t-1}$
88	$[1,2,4,4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^8+t^7+2\,t^6+2\,t^5+2\,t^4+2\,t^3+t^2+t-1}]$
89	$[1,3,3,1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-t^5-t^4-2\,t+1}$
90	$[1,3,3,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-3\,t^5-2\,t^4-2\,t^3-t^2-t+1}$
91	$[1,3,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{2\,t^7+t^6-2\,t^5-t^4-t^3-2\,t^2-t+1}$
92	$[1,3,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-2\,t^5-t^3-2\,t+1}$
93	$[1,3,4,1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^6-t^5-t^4-2\,t+1}$
94	$[1,3,4,2]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9-t^7-t^6-3\,t^5-2\,t^4-2\,t^3-t^2-t+1}$
95	$[1,3,4,3]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)^2}{t^7-2\,t^5-t^4-t^3-2\,t^2-t+1}$
96	$[1,3,4,4]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^6-2\,t^5-t^3-2\,t+1}$
97	$[1,4,4,1]$	$-\frac{\left(t^2+t+1\right)(t+1)^2}{t^3+2\,t^2+t-1}$
98	$[1,4,4,2]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)^2}{t^6+3\,t^5+4\,t^4+4\,t^3+2\,t^2-1}$
99	$[1,4,4,3]$	$-\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)^2}{t^6+2\,t^5+t^4+t^3+2\,t^2+t-1}$
100	$[1,4,4,4]$	$-\frac{\left(t^2+t+1\right)(t+1)^2}{2\,t^3+2\,t^2+t-1}$

	$[r1, r2, r3, r4]$	*Poincaré series*
101	$[2,1,1,1]$	$\frac{(t^2+t+1)(t^3+1)(t^3+t^2+t+1)(t+1)}{t^8+t^6-t^4-t^2-t+1}$
102	$[2,1,1,2]$	$\frac{(t^2+t+1)(t^3+1)(t^3+t^2+t+1)(t+1)}{t^8+t^6-t^5-t^3-t^2-t+1}$
103	$[2,1,1,3]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)(t+1)}{t^8+t^6-t^5-t^4-t^3-t^2-t+1}$
104	$[2,1,1,4]$	$-\frac{(t^2+t+1)(t^3+1)(t^3+t^2+t+1)(t+1)}{t^5+t^4+t^3+t^2+t-1}$
105	$[2,1,2,1]$	$\frac{(t^2+t+1)(t^3+1)(t^3+t^2+t+1)(t+1)}{t^9+t^8-t^4-t^3-t^2-t+1}$
106	$[2,1,2,2]$	$\frac{(t^2+t+1)(t^3+1)(t^3+t^2+t+1)(t+1)}{t^9+t^8-t^5-2t^3-t^2-t+1}$
107	$[2,1,2,3]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)(t+1)}{t^9+t^8-t^5-t^4-2t^3-t^2-t+1}$
108	$[2,1,2,4]$	$\frac{(t^2+t+1)(t^3+1)(t^3+t^2+t+1)(t+1)}{t^9-t^6-t^5-t^4-2t^3-t^2-t+1}$
109	$[2,1,3,1]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)(t+1)}{t^9+t^8-t^5-2t^4-t^3-t^2-t+1}$
110	$[2,1,3,2]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)(t+1)}{t^9+t^8-2t^5-t^4-2t^3-t^2-t+1}$
111	$[2,1,3,3]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)(t+1)}{t^9+t^8-2t^5-2t^4-2t^3-t^2-t+1}$
112	$[2,1,3,4]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)(t+1)}{t^9-t^6-2t^5-2t^4-2t^3-t^2-t+1}$
113	$[2,1,4,1]$	$-\frac{(t^2+t+1)(t^3+1)(t^3+t^2+t+1)(t+1)}{t^7+t^6+t^5+2t^4+t^3+t^2+t-1}$
114	$[2,1,4,2]$	$-\frac{(t^2+t+1)(t^3+1)(t^3+t^2+t+1)(t+1)}{t^7+t^6+2t^5+t^4+2t^3+t^2+t-1}$
115	$[2,1,4,3]$	$-\frac{(t^3+t^2+t+1)(t^5+t^4+t^3+t^2+t+1)(t+1)}{t^7+t^6+2t^5+2t^4+2t^3+t^2+t-1}$
116	$[2,1,4,4]$	$-\frac{(t^3+t^2+t+1)(t^2+t+1)(t^3+1)(t+1)}{t^8+t^7+2t^6+2t^5+2t^4+2t^3+t^2+t-1}$
117	$[2,2,2,1]$	$\frac{(t^3+t^2+t+1)(t^2+t+1)(t+1)}{2t^6+t^5-3t^3-t^2-t+1}$
118	$[2,2,2,2]$	$\frac{(t^3+t^2+t+1)(t+1)^2}{2t^5+t^4-2t^3-2t^2-t+1}$
119	$[2,2,2,3]$	$\frac{(t^3+t^2+t+1)(t^5+t^4+t^3+t^2+t+1)(t+1)}{2t^9+t^8-t^6-t^5-t^4-3t^3-t^2-t+1}$
120	$[2,2,2,4]$	$\frac{(t^3+t^2+t+1)(t+1)}{2t^4-2t^3-2t+1}$

	$[r1, r2, r3, r4]$	*Poincaré series*
121	$[2, 2, 3, 1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{2\,t^8-t^7+t^6-2\,t^5+t^4-3\,t^3+t^2-2\,t+1}$
122	$[2, 2, 3, 2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-t^6-2\,t^5-t^4-3\,t^3-t^2-t+1}$
123	$[2, 2, 3, 3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-t^6-2\,t^5-2\,t^4-3\,t^3-t^2-t+1}$
124	$[2, 2, 3, 4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+2\,t^6-4\,t^5+2\,t^4-4\,t^3+t^2-2\,t+1}$
125	$[2, 2, 4, 1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^4-3\,t^3-t^2-t+1}$
126	$[2, 2, 4, 2]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)^2}{t^5-2\,t^3-2\,t^2-t+1}$
127	$[2, 2, 4, 3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-2\,t^5-2\,t^4-3\,t^3-t^2-t+1}$
128	$[2, 2, 4, 4]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{t^4-2\,t^3-2\,t+1}$
129	$[2, 3, 3, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-t^7+t^6-2\,t^5-3\,t^3+t^2-2\,t+1}$
130	$[2, 3, 3, 2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-t^6-3\,t^5-2\,t^4-3\,t^3-t^2-t+1}$
131	$[2, 3, 3, 3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-t^6-3\,t^5-3\,t^4-3\,t^3-t^2-t+1}$
132	$[2, 3, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+2\,t^6-4\,t^5+t^4-4\,t^3+t^2-2\,t+1}$
133	$[2, 3, 4, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-t^7-2\,t^5-3\,t^3+t^2-2\,t+1}$
134	$[2, 3, 4, 2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-3\,t^5-2\,t^4-3\,t^3-t^2-t+1}$
135	$[2, 3, 4, 3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-3\,t^5-3\,t^4-3\,t^3-t^2-t+1}$
136	$[2, 3, 4, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7+t^6-4\,t^5+t^4-4\,t^3+t^2-2\,t+1}$
137	$[2, 4, 4, 1]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^5+2\,t^4+3\,t^3+t^2+t-1}$
138	$[2, 4, 4, 2]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)^2}{t^4+2\,t^3+2\,t^2+t-1}$
139	$[2, 4, 4, 3]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^8+2\,t^7+3\,t^6+3\,t^5+3\,t^4+3\,t^3+t^2+t-1}$
140	$[2, 4, 4, 4]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)}{2\,t^3+2\,t-1}$

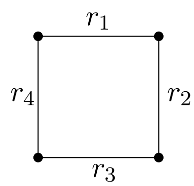
	$[r1, r2, r3, r4]$	*Poincaré series*
141	$[3, 1, 1, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{2\,t^6-t^5-t^4+t^3-2\,t+1}$
142	$[3, 1, 1, 2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{2\,t^9+t^8+t^6-2\,t^5-t^4-t^3-t^2-t+1}$
143	$[3, 1, 1, 3]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)(t+1)^2}{2\,t^7+t^6-2\,t^5-2\,t^2-t+1}$
144	$[3, 1, 1, 4]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{2\,t^6-2\,t^5-2\,t+1}$
145	$[3, 1, 2, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-t^7+t^6-t^5-2\,t^3+t^2-2\,t+1}$
146	$[3, 1, 2, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-2\,t^5-t^4-2\,t^3-t^2-t+1}$
147	$[3, 1, 2, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-2\,t^5-2\,t^4-2\,t^3-t^2-t+1}$
148	$[3, 1, 2, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+2\,t^6-3\,t^5+t^4-3\,t^3+t^2-2\,t+1}$
149	$[3, 1, 3, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-t^5-t^4-2\,t+1}$
150	$[3, 1, 3, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-3\,t^5-2\,t^4-2\,t^3-t^2-t+1}$
151	$[3, 1, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{2\,t^7+t^6-2\,t^5-t^4-t^3-2\,t^2-t+1}$
152	$[3, 1, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-2\,t^5-t^3-2\,t+1}$
153	$[3, 1, 4, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-t^5-t^4-2\,t+1}$
154	$[3, 1, 4, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9-t^7-t^6-3\,t^5-2\,t^4-2\,t^3-t^2-t+1}$
155	$[3, 1, 4, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{t^7-2\,t^5-t^4-t^3-2\,t^2-t+1}$
156	$[3, 1, 4, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-t^3-2\,t+1}$
157	$[3, 2, 2, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{2\,t^8-t^7+t^6-2\,t^5+t^4-3\,t^3+t^2-2\,t+1}$
158	$[3, 2, 2, 2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-t^6-2\,t^5-t^4-3\,t^3-t^2-t+1}$
159	$[3, 2, 2, 3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-t^6-2\,t^5-2\,t^4-3\,t^3-t^2-t+1}$
160	$[3, 2, 2, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+2\,t^6-4\,t^5+2\,t^4-4\,t^3+t^2-2\,t+1}$

	$[r1, r2, r3, r4]$	*Poincaré series*
161	[3, 2, 3, 1]	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-t^7+t^6-2\,t^5-3\,t^3+t^2-2\,t+1}$
162	[3, 2, 3, 2]	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-t^6-3\,t^5-2\,t^4-3\,t^3-t^2-t+1}$
163	[3, 2, 3, 3]	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-t^6-3\,t^5-3\,t^4-3\,t^3-t^2-t+1}$
164	[3, 2, 3, 4]	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+2\,t^6-4\,t^5+t^4-4\,t^3+t^2-2\,t+1}$
165	[3, 2, 4, 1]	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{t^8-t^7-2\,t^5-3\,t^3+t^2-2\,t+1}$
166	[3, 2, 4, 2]	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-3\,t^5-2\,t^4-3\,t^3-t^2-t+1}$
167	[3, 2, 4, 3]	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-3\,t^5-3\,t^4-3\,t^3-t^2-t+1}$
168	[3, 2, 4, 4]	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7+t^6-4\,t^5+t^4-4\,t^3+t^2-2\,t+1}$
169	[3, 3, 3, 1]	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-t^5-t^4-t^3-2\,t+1}$
170	[3, 3, 3, 2]	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-t^6-4\,t^5-3\,t^4-3\,t^3-t^2-t+1}$
171	[3, 3, 3, 3]	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{2\,t^7+t^6-2\,t^5-2\,t^4-2\,t^3-2\,t^2-t+1}$
172	[3, 3, 3, 4]	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-2\,t^5-2\,t^3-2\,t+1}$
173	[3, 3, 4, 1]	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-t^5-t^4-t^3-2\,t+1}$
174	[3, 3, 4, 2]	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-4\,t^5-3\,t^4-3\,t^3-t^2-t+1}$
175	[3, 3, 4, 3]	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{t^7-2\,t^5-2\,t^4-2\,t^3-2\,t^2-t+1}$
176	[3, 3, 4, 4]	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-2\,t^3-2\,t+1}$
177	[3, 4, 4, 1]	$-\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^5+t^4+t^3+2\,t-1}$
178	[3, 4, 4, 2]	$-\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^8+2\,t^7+3\,t^6+4\,t^5+3\,t^4+3\,t^3+t^2+t-1}$
179	[3, 4, 4, 3]	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{t^6+2\,t^5+2\,t^4+2\,t^3+2\,t^2+t-1}$
180	[3, 4, 4, 4]	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^5+2\,t^3+2\,t-1}$

	$[r1, r2, r3, r4]$	*Poincaré series*
181	$[4, 1, 1, 1]$	$\frac{\left(t^2+t+1\right)\left(t+1\right)^2}{t^4-2\,t^2-t+1}$
182	$[4, 1, 1, 2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)\left(t+1\right)^2}{t^7+t^6-t^5-2\,t^4-3\,t^3-2\,t^2+1}$
183	$[4, 1, 1, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t+1\right)^2}{t^7-2\,t^5-2\,t^2-t+1}]$
184	$[4, 1, 1, 4]$	$\frac{\left(t^2+t+1\right)\left(t+1\right)^2}{t^4-t^3-2\,t^2-t+1}$
185	$[4, 1, 2, 1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)\left(t+1\right)}{t^6-t^4-2\,t^3-t^2-t+1}$
186	$[4, 1, 2, 2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)\left(t+1\right)^2}{t^7+t^6-t^5-3\,t^4-4\,t^3-2\,t^2+1}$
187	$[4, 1, 2, 3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)\left(t+1\right)}{t^9-t^7-t^6-2\,t^5-2\,t^4-2\,t^3-t^2-t+1}$
188	$[4, 1, 2, 4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)\left(t+1\right)}{t^6-t^5-t^4-3\,t^3-t^2-t+1}$
189	$[4, 1, 3, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t+1\right)}{t^6-t^5-t^4-2\,t+1}$
190	$[4, 1, 3, 2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)\left(t+1\right)}{t^9-t^7-t^6-3\,t^5-2\,t^4-2\,t^3-t^2-t+1}$
191	$[4, 1, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t+1\right)^2}{t^7-2\,t^5-t^4-t^3-2\,t^2-t+1}$
192	$[4, 1, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t+1\right)}{t^6-2\,t^5-t^3-2\,t+1}$
193	$[4, 1, 4, 1]$	$-\frac{\left(t^2+t+1\right)\left(t+1\right)^2}{t^3+2\,t^2+t-1}$
194	$[4, 1, 4, 2]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)\left(t+1\right)^2}{t^6+3\,t^5+4\,t^4+4\,t^3+2\,t^2-1}$
195	$[4, 1, 4, 3]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t+1\right)^2}{t^6+2\,t^5+t^4+t^3+2\,t^2+t-1}$
196	$[4, 1, 4, 4]$	$-\frac{\left(t^2+t+1\right)\left(t+1\right)^2}{2\,t^3+2\,t^2+t-1}$
197	$[4, 2, 2, 1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)\left(t+1\right)}{t^6-t^4-3\,t^3-t^2-t+1}$
198	$[4, 2, 2, 2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t+1\right)^2}{t^5-2\,t^3-2\,t^2-t+1}$
199	$[4, 2, 2, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t+1\right)}{t^9-t^7-2\,t^6-2\,t^5-2\,t^4-3\,t^3-t^2-t+1}]$
200	$[4, 2, 2, 4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t+1\right)}{t^4-2\,t^3-2\,t+1}$

	$[r1, r2, r3, r4]$	*Poincaré series*
201	$[4, 2, 3, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-t^7-2\,t^5-3\,t^3+t^2-2\,t+1}$
202	$[4, 2, 3, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-3\,t^5-2\,t^4-3\,t^3-t^2-t+1}]$
203	$[4, 2, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-3\,t^5-3\,t^4-3\,t^3-t^2-t+1}]$
204	$[4, 2, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7+t^6-4\,t^5+t^4-4\,t^3+t^2-2\,t+1}$
205	$[4, 2, 4, 1]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^5+2\,t^4+3\,t^3+t^2+t-1}$
206	$[4, 2, 4, 2]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)^2}{t^4+2\,t^3+2\,t^2+t-1}$
207	$[4, 2, 4, 3]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^8+2\,t^7+3\,t^6+3\,t^5+3\,t^4+3\,t^3+t^2+t-1}$
208	$[4, 2, 4, 4]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)}{2\,t^3+2\,t-1}$
209	$[4, 3, 3, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-t^5-t^4-t^3-2\,t+1}$
210	$[4, 3, 3, 2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-4\,t^5-3\,t^4-3\,t^3-t^2-t+1}$
211	$[4, 3, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{t^7-2\,t^5-2\,t^4-2\,t^3-2\,t^2-t+1}$
212	$[4, 3, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-2\,t^3-2\,t+1}$
213	$[4, 3, 4, 1]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^5+t^4+t^3+2\,t-1}$
214	$[4, 3, 4, 2]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^8+2\,t^7+3\,t^6+4\,t^5+3\,t^4+3\,t^3+t^2+t-1}$
215	$[4, 3, 4, 3]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{t^6+2\,t^5+2\,t^4+2\,t^3+2\,t^2+t-1}$
216	$[4, 3, 4, 4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^5+2\,t^3+2\,t-1}$
217	$[4, 4, 4, 1]$	$-\frac{\left(t^2+t+1\right)(t+1)^2}{t^4+2\,t^3+2\,t^2+t-1}$
218	$[4, 4, 4, 2]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)^2}{t^5+2\,t^4+2\,t^3+2\,t^2+t-1}$
219	$[4, 4, 4, 3]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)^2}{t^7+2\,t^6+2\,t^5+2\,t^4+2\,t^3+2\,t^2+t-1}$
220	$[4, 4, 4, 4]$	$-\frac{(t+1)^2}{t^2+2\,t-1}$

The Poincaré series of Cartan matrices with reduced Coxeter graph 4-IV

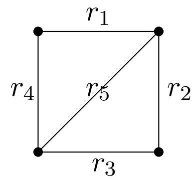


	$[r1,r2,r3,r4]$	*Poincaré series*
221	$[1,1,1,1]$	$-\frac{t^3+t^2+t+1}{t^3-3\,t^2+3\,t-1}$
222	$[1,1,1,2]$	$-\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^7-2\,t^6+2\,t^4-2\,t^3+2\,t-1}$
223	$[1,1,1,3]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8+t^6-t^5-t^3+t^2-2\,t+1}$
224	$[1,1,1,4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^3+t^2+t-1}$
225	$[1,1,2,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^6-t^4+t^3-2\,t+1}$
226	$[1,1,2,3]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9+t^8+t^7-t^5-t^4-t^3-t^2-t+1}$
227	$[1,1,2,4]$	$-\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^6+t^5+t^4+t^3+t^2+t-1}$
228	$[1,1,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)\left(t^2+1\right)}{2\,t^8-t^7+2\,t^6-2\,t^5-2\,t^3+t^2-2\,t+1}$
229	$[1,1,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)\left(t^2+1\right)}{t^8-t^7+t^6-2\,t^5-2\,t^3+t^2-2\,t+1}$
230	$[1,1,4,4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^5+t^4+2\,t^3+t^2+t-1}$
231	$[1,2,1,2]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^7-2\,t^6+t^4-t^3+2\,t-1}$
232	$[1,2,1,3]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9+t^8+t^7+t^6-t^5-t^4-t^3-t^2-t+1}$
233	$[1,2,1,4]$	$-\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^5+t^4+t^3+t^2+t-1}$
234	$[1,2,2,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^7-2\,t+1}$
235	$[1,2,2,3]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8+t^7-t^5-t^4-2\,t^3-t^2-t+1}$
236	$[1,2,2,4]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9-t^6-t^5-t^4-2\,t^3-t^2-t+1}$
237	$[1,2,3,2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9+t^8+t^7-t^5-t^4-2\,t^3-t^2-t+1}$
238	$[1,2,3,3]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{2\,t^9+t^8+t^7-2\,t^5-2\,t^4-2\,t^3-t^2-t+1}]$
239	$[1,2,3,4]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9-t^6-2\,t^5-2\,t^4-2\,t^3-t^2-t+1}$
240	$[1,2,4,2]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^4+2\,t-1}$

	$[r1, r2, r3, r4]$	*Poincaré series*
241	$[1,2,4,3]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9-t^6-2\,t^5-2\,t^4-2\,t^3-t^2-t+1}$
242	$[1,2,4,4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^8+t^7+2\,t^6+2\,t^5+2\,t^4+2\,t^3+t^2+t-1}$
243	$[1,3,1,3]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{3\,t^6-2\,t^5-2\,t+1}$
244	$[1,3,1,4]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{2\,t^6-2\,t^5-2\,t+1}$
245	$[1,3,2,3]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)(t+1)\left(t^2+1\right)}{3\,t^8-2\,t^7+3\,t^6-3\,t^5+t^4-3\,t^3+t^2-2\,t+1}$
246	$[1,3,2,4]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)(t+1)\left(t^2+1\right)}{2\,t^8-2\,t^7+2\,t^6-3\,t^5+t^4-3\,t^3+t^2-2\,t+1}$
247	$[1,3,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{3\,t^6-2\,t^5-t^3-2\,t+1}$
248	$[1,3,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-2\,t^5-t^3-2\,t+1}$
249	$[1,3,4,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-2\,t^5-t^3-2\,t+1}$
250	$[1,3,4,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-t^3-2\,t+1}$
251	$[1,4,1,4]$	$\frac{\left(t^2+t+1\right)(t+1)^2}{t^4-t^3-2\,t^2-t+1}$
252	$[1,4,2,4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^5-t^4-3\,t^3-t^2-t+1}$
253	$[1,4,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-t^3-2\,t+1}$
254	$[1,4,4,4]$	$-\frac{\left(t^2+t+1\right)(t+1)^2}{2\,t^3+2\,t^2+t-1}$
255	$[2,2,2,2]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{3\,t^4-2\,t^3-2\,t+1}$
256	$[2,2,2,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+3\,t^6-4\,t^5+3\,t^4-4\,t^3+t^2-2\,t+1}$
257	$[2,2,2,4]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{2\,t^4-2\,t^3-2\,t+1}$
258	$[2,2,3,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+3\,t^6-4\,t^5+2\,t^4-4\,t^3+t^2-2\,t+1}$
259	$[2,2,3,4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+2\,t^6-4\,t^5+2\,t^4-4\,t^3+t^2-2\,t+1}$
260	$[2,2,4,4]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{t^4-2\,t^3-2\,t+1}$

	$[r1, r2, r3, r4]$	*Poincaré series*
261	$[2,3,2,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+3\,t^6-4\,t^5+2\,t^4-4\,t^3+t^2-2\,t+1}$
262	$[2,3,2,4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+2\,t^6-4\,t^5+2\,t^4-4\,t^3+t^2-2\,t+1}$
263	$[2,3,3,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+3\,t^6-4\,t^5+t^4-4\,t^3+t^2-2\,t+1}$
264	$[2,3,3,4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+2\,t^6-4\,t^5+t^4-4\,t^3+t^2-2\,t+1}$
265	$[2,3,4,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+2\,t^6-4\,t^5+t^4-4\,t^3+t^2-2\,t+1}$
266	$[2,3,4,4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{t^8-2\,t^7+t^6-4\,t^5+t^4-4\,t^3+t^2-2\,t+1}$
267	$[2,4,2,4]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{t^4-2\,t^3-2\,t+1}$
268	$[2,4,3,4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{t^8-2\,t^7+t^6-4\,t^5+t^4-4\,t^3+t^2-2\,t+1}$
269	$[2,4,4,4]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)}{2\,t^3+2\,t-1}$
270	$[3,3,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{3\,t^6-2\,t^5-2\,t^3-2\,t+1}$
271	$[3,3,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-2\,t^5-2\,t^3-2\,t+1}$
272	$[3,3,4,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-2\,t^3-2\,t+1}$
273	$[3,4,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-2\,t^3-2\,t+1}$
274	$[3,4,4,4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^5+2\,t^3+2\,t-1}$
275	$[4,4,4,4]$	$-\frac{(t+1)^2}{t^2+2\,t-1}$

The Poincaré series of Cartan matrices with reduced Coxeter graph 4-V



	$[r1, r2, r3, r4, r5]$	*Poincaré series*
276	$[1, 1, 1, 1, 1]$	$\frac{(t^2+t+1)(t^3+t^2+t+1)}{t^5-2t+1}$
277	$[1, 1, 1, 1, 2]$	$\frac{(t^3+t^2+t+1)(t^2+t+1)(t+1)}{t^6+t^5-t^3-2t^2-t+1}$
278	$[1, 1, 1, 1, 3]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{t^8-2t^4-2t+1}$
279	$[1, 1, 1, 1, 4]$	$-\frac{(t^2+t+1)(t^3+t^2+t+1)}{t^3+2t-1}$
280	$[1, 1, 1, 2, 1]$	$\frac{(t^3+t^2+t+1)(t^5+t^4+t^3+t^2+t+1)(t+1)}{t^9+t^8+t^6-t^5-t^4-2t^2-t+1}$
281	$[1, 1, 1, 2, 2]$	$\frac{(t^3+t^2+t+1)(t^5+t^4+t^3+t^2+t+1)(t+1)}{t^9+t^8-t^5-t^4-t^3-2t^2-t+1}$
282	$[1, 1, 1, 2, 3]$	$\frac{(t^3+t^2+t+1)(t^5+t^4+t^3+t^2+t+1)(t+1)}{t^9+t^8-2t^5-2t^4-t^3-2t^2-t+1}$
283	$[1, 1, 1, 2, 4]$	$-\frac{(t^3+t^2+t+1)(t^5+t^4+t^3+t^2+t+1)(t+1)}{t^7+t^6+2t^5+2t^4+t^3+2t^2+t-1}$
284	$[1, 1, 1, 3, 1]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)(t^2+1)}{2t^8-t^7+t^6-2t^4-2t+1}$
285	$[1, 1, 1, 3, 2]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)(t^2+1)}{2t^8-t^7+t^6-t^5-t^4-t^3-2t+1}$
286	$[1, 1, 1, 3, 3]$	$\frac{(t^3+1)(t^2+t+1)(t+1)(t^2+1)}{2t^8-t^7+t^6-t^5-2t^4-t^3-2t+1}$
287	$[1, 1, 1, 3, 4]$	$\frac{(t^3+1)(t^2+t+1)(t+1)(t^2+1)}{t^8-t^7-t^5-2t^4-t^3-2t+1}$
288	$[1, 1, 1, 4, 1]$	$\frac{(t^2+t+1)(t^3+t^2+t+1)(t+1)}{t^6-t^4-t^3-2t^2-t+1}$
289	$[1, 1, 1, 4, 2]$	$\frac{(t^2+t+1)(t^3+t^2+t+1)(t+1)}{t^6-t^4-2t^3-2t^2-t+1}$
290	$[1, 1, 1, 4, 3]$	$\frac{(t^2+t+1)(t^3+1)(t^3+t^2+t+1)}{t^8-t^7-t^5-2t^4-t^3-2t+1}$

	$[r1, r2, r3, r4, r5]$	*Poincaré series*
291	$[1,1,1,4,4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^5+2\,t^4+2\,t^3+2\,t^2+t-1}$
292	$[1,1,2,2,1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2\,t^6+t^5-2\,t^3-2\,t^2-t+1}$
293	$[1,1,2,2,2]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2\,t^6+t^5-3\,t^3-2\,t^2-t+1}$
294	$[1,1,2,2,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-t^7+t^6-2\,t^5-2\,t^3-2\,t+1}$
295	$[1,1,2,2,4]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^6-t^4-3\,t^3-2\,t^2-t+1}$
296	$[1,1,2,3,1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-t^7+t^6-t^5-t^4-t^3-2\,t+1}$
297	$[1,1,2,3,2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-t^7+t^6-2\,t^5-2\,t^3-2\,t+1}$
298	$[1,1,2,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-t^7+t^6-2\,t^5-t^4-2\,t^3-2\,t+1}$
299	$[1,1,2,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-t^7-2\,t^5-t^4-2\,t^3-2\,t+1}$
300	$[1,1,2,4,1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^4-2\,t^3-2\,t^2-t+1}$
301	$[1,1,2,4,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^4-3\,t^3-2\,t^2-t+1}$
302	$[1,1,2,4,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-t^7-2\,t^5-t^4-2\,t^3-2\,t+1}$
303	$[1,1,2,4,4]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^5+2\,t^4+3\,t^3+2\,t^2+t-1}$
304	$[1,1,3,3,1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-t^7+t^6-t^5-2\,t^4-t^3-2\,t+1}$
305	$[1,1,3,3,2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-t^7+t^6-2\,t^5-t^4-2\,t^3-2\,t+1}$
306	$[1,1,3,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-t^7+t^6-2\,t^5-2\,t^4-2\,t^3-2\,t+1}$
307	$[1,1,3,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-t^7-2\,t^5-2\,t^4-2\,t^3-2\,t+1}$
308	$[1,1,3,4,1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)\left(t^3+t^2+t+1\right)}{t^8-t^7-t^5-2\,t^4-t^3-2\,t+1}$
309	$[1,1,3,4,2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-t^7-2\,t^5-t^4-2\,t^3-2\,t+1}$
310	$[1,1,3,4,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-t^7-2\,t^5-2\,t^4-2\,t^3-2\,t+1}$

	$[r1, r2, r3, r4, r5]$	*Poincaré series*
311	$[1,1,3,4,4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+1\right)\left(t^3+t^2+t+1\right)}{t^7+t^6+2\,t^5+2\,t^4+2\,t^3+2\,t-1}$
312	$[1,1,4,4,1]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^5+2\,t^4+2\,t^3+2\,t^2+t-1}$
313	$[1,1,4,4,2]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^5+2\,t^4+3\,t^3+2\,t^2+t-1}$
314	$[1,1,4,4,3]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^7+t^6+2\,t^5+2\,t^4+2\,t^3+2\,t-1}$
315	$[1,1,4,4,4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^6+2\,t^5+3\,t^4+3\,t^3+2\,t^2+t-1}$
316	$[1,2,1,2,1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9+t^8+t^6-t^5-t^4-t^3-2\,t^2-t+1}$
317	$[1,2,1,2,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9+t^8-t^5-t^4-2\,t^3-2\,t^2-t+1}]$
318	$[1,2,1,2,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9+t^8-2\,t^5-2\,t^4-2\,t^3-2\,t^2-t+1}$
319	$[1,2,1,2,4]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^7+t^6+2\,t^5+2\,t^4+2\,t^3+2\,t^2+t-1}$
320	$[1,2,1,3,1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8+t^6-2\,t^5-2\,t^4-t^3-2\,t^2-t+1}$
321	$[1,2,1,3,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-2\,t^5-2\,t^4-2\,t^3-2\,t^2-t+1}$
322	$[1,2,1,3,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-3\,t^5-3\,t^4-2\,t^3-2\,t^2-t+1}$
323	$[1,2,1,3,4]$	$\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^9-t^7-t^6-3\,t^5-3\,t^4-2\,t^3-2\,t^2-t+1}$
324	$[1,2,1,4,1]$	$\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^5-2\,t^4-t^3-2\,t^2-t+1}$
325	$[1,2,1,4,2]$	$\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^9-t^7-t^6-2\,t^5-2\,t^4-2\,t^3-2\,t^2-t+1}$
326	$[1,2,1,4,3]$	$\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^9-t^7-t^6-3\,t^5-3\,t^4-2\,t^3-2\,t^2-t+1}$
327	$[1,2,1,4,4]$	$-\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^8+2\,t^7+2\,t^6+3\,t^5+3\,t^4+2\,t^3+2\,t^2+t-1}$
328	$[1,2,2,1,1]$	$\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^9+t^8+t^6-t^5-t^4-t^3-2\,t^2-t+1}$
329	$[1,2,2,1,2]$	$\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^9+t^8-t^5-t^4-2\,t^3-2\,t^2-t+1}$
330	$[1,2,2,1,3]$	$\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^9+t^8-2\,t^5-2\,t^4-2\,t^3-2\,t^2-t+1}$

	$[r1, r2, r3, r4, r5]$	*Poincaré series*
331	$[1, 2, 2, 1, 4]$	$-\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^7+t^6+2t^5+2t^4+2t^3+2t^2+t-1}$
332	$[1, 2, 2, 2, 1]$	$\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2t^9+t^8-t^5-t^4-2t^3-2t^2-t+1}$
333	$[1, 2, 2, 2, 2]$	$\frac{\left(t^3+1\right)\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2t^9+t^8-t^6-t^5-t^4-3t^3-2t^2-t+1}$
334	$[1, 2, 2, 2, 3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2t^9+t^8-t^6-2t^5-2t^4-3t^3-2t^2-t+1}$
335	$[1, 2, 2, 2, 4]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9-t^7-2t^6-2t^5-2t^4-3t^3-2t^2-t+1}$
336	$[1, 2, 2, 3, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2t^9+t^8-2t^5-2t^4-2t^3-2t^2-t+1}$
337	$[1, 2, 2, 3, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2t^9+t^8-t^6-2t^5-2t^4-3t^3-2t^2-t+1}$
338	$[1, 2, 2, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2t^9+t^8-t^6-3t^5-3t^4-3t^3-2t^2-t+1}$
339	$[1, 2, 2, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2t^6-3t^5-3t^4-3t^3-2t^2-t+1}$
340	$[1, 2, 2, 4, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9-t^7-t^6-2t^5-2t^4-2t^3-2t^2-t+1}$
341	$[1, 2, 2, 4, 2]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9-t^7-2t^6-2t^5-2t^4-3t^3-2t^2-t+1}$
342	$[1, 2, 2, 4, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2t^6-3t^5-3t^4-3t^3-2t^2-t+1}$
343	$[1, 2, 2, 4, 4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^8+2t^7+3t^6+3t^5+3t^4+3t^3+2t^2+t-1}$
344	$[1, 2, 3, 1, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{2t^9+t^8+t^6-2t^5-2t^4-t^3-2t^2-t+1}$
345	$[1, 2, 3, 1, 2]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{2t^9+t^8-2t^5-2t^4-2t^3-2t^2-t+1}$
346	$[1, 2, 3, 1, 3]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{2t^9+t^8-3t^5-3t^4-2t^3-2t^2-t+1}$
347	$[1, 2, 3, 1, 4]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9-t^7-t^6-3t^5-3t^4-2t^3-2t^2-t+1}$
348	$[1, 2, 3, 2, 1]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2t^9+t^8-2t^5-2t^4-2t^3-2t^2-t+1}$
349	$[1, 2, 3, 2, 2]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2t^9+t^8-t^6-2t^5-2t^4-3t^3-2t^2-t+1}$
350	$[1, 2, 3, 2, 3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2t^9+t^8-t^6-3t^5-3t^4-3t^3-2t^2-t+1}$

	$[r1, r2, r3, r4, r5]$	*Poincaré series*
351	$[1,2,3,2,4]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-3\,t^5-3\,t^4-3\,t^3-2\,t^2-t+1}$
352	$[1,2,3,3,1]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-3\,t^5-3\,t^4-2\,t^3-2\,t^2-t+1}$
353	$[1,2,3,3,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-t^6-3\,t^5-3\,t^4-3\,t^3-2\,t^2-t+1}$
354	$[1,2,3,3,3]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{2\,t^9+t^8-t^6-4\,t^5-4\,t^4-3\,t^3-2\,t^2-t+1}$
355	$[1,2,3,3,4]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-4\,t^5-4\,t^4-3\,t^3-2\,t^2-t+1}$
356	$[1,2,3,4,1]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9-t^7-t^6-3\,t^5-3\,t^4-2\,t^3-2\,t^2-t+1}$
357	$[1,2,3,4,2]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-3\,t^5-3\,t^4-3\,t^3-2\,t^2-t+1}$
358	$[1,2,3,4,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-4\,t^5-4\,t^4-3\,t^3-2\,t^2-t+1}$
359	$[1,2,3,4,4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^8+2\,t^7+3\,t^6+4\,t^5+4\,t^4+3\,t^3+2\,t^2+t-1}$
360	$[1,2,4,1,1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9-t^7-2\,t^5-2\,t^4-t^3-2\,t^2-t+1}$
361	$[1,2,4,1,2]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9-t^7-t^6-2\,t^5-2\,t^4-2\,t^3-2\,t^2-t+1}$
362	$[1,2,4,1,3]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9-t^7-t^6-3\,t^5-3\,t^4-2\,t^3-2\,t^2-t+1}$
363	$[1,2,4,1,4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^8+2\,t^7+2\,t^6+3\,t^5+3\,t^4+2\,t^3+2\,t^2+t-1}$
364	$[1,2,4,2,1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9-t^7-t^6-2\,t^5-2\,t^4-2\,t^3-2\,t^2-t+1}$
365	$[1,2,4,2,2]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9-t^7-2\,t^6-2\,t^5-2\,t^4-3\,t^3-2\,t^2-t+1}$
366	$[1,2,4,2,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-3\,t^5-3\,t^4-3\,t^3-2\,t^2-t+1}$
367	$[1,2,4,2,4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^8+2\,t^7+3\,t^6+3\,t^5+3\,t^4+3\,t^3+2\,t^2+t-1}$
368	$[1,2,4,3,1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^9-t^7-t^6-3\,t^5-3\,t^4-2\,t^3-2\,t^2-t+1}$
369	$[1,2,4,3,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-3\,t^5-3\,t^4-3\,t^3-2\,t^2-t+1}$
370	$[1,2,4,3,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^9-t^7-2\,t^6-4\,t^5-4\,t^4-3\,t^3-2\,t^2-t+1}$

	$[r1,r2,r3,r4,r5]$	*Poincaré series*
371	$[1,2,4,3,4]$	$-\frac{(t^3+t^2+t+1)(t^5+t^4+t^3+t^2+t+1)(t+1)}{t^8+2t^7+3t^6+4t^5+4t^4+3t^3+2t^2+t-1}$
372	$[1,2,4,4,1]$	$-\frac{(t^2+t+1)(t^3+t^2+t+1)(t^3+1)(t+1)}{t^8+2t^7+2t^6+3t^5+3t^4+2t^3+2t^2+t-1}$
373	$[1,2,4,4,2]$	$-\frac{(t^2+t+1)(t^3+t^2+t+1)(t^3+1)(t+1)}{t^8+2t^7+3t^6+3t^5+3t^4+3t^3+2t^2+t-1}$
374	$[1,2,4,4,3]$	$-\frac{(t^3+t^2+t+1)(t^5+t^4+t^3+t^2+t+1)(t+1)}{t^8+2t^7+3t^6+4t^5+4t^4+3t^3+2t^2+t-1}$
375	$[1,2,4,4,4]$	$-\frac{(t^2+t+1)(t^3+t^2+t+1)(t^3+1)(t+1)}{t^9+2t^8+3t^7+4t^6+4t^5+4t^4+3t^3+2t^2+t-1}$
376	$[1,3,1,3,1]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)}{3t^6-2t^5-t^4+t^3-t^2-2t+1}$
377	$[1,3,1,3,2]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{3t^8-2t^7+2t^6-2t^5-t^4-2t^3-2t+1}$
378	$[1,3,1,3,3]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)}{3t^6-2t^5-t^4-t^2-2t+1}$
379	$[1,3,1,3,4]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)}{2t^6-2t^5-t^4-t^2-2t+1}$
380	$[1,3,1,4,1]$	$\frac{(t^2+t+1)(t^3+1)(t+1)}{2t^6-2t^5-t^4+t^3-t^2-2t+1}$
381	$[1,3,1,4,2]$	$\frac{(t^2+t+1)(t^3+t^2+t+1)(t^3+1)}{2t^8-2t^7+t^6-2t^5-t^4-2t^3-2t+1}$
382	$[1,3,1,4,3]$	$\frac{(t^2+t+1)(t^3+1)(t+1)}{2t^6-2t^5-t^4-t^2-2t+1}$
383	$[1,3,1,4,4]$	$\frac{(t^2+t+1)(t^3+1)(t+1)}{t^6-2t^5-t^4-t^2-2t+1}$
384	$[1,3,2,2,1]$	$\frac{(t^2+t+1)(t^3+t^2+t+1)(t^3+1)}{3t^8-2t^7+2t^6-2t^5-2t^3-2t+1}$
385	$[1,3,2,2,2]$	$\frac{(t^2+t+1)(t^3+t^2+t+1)(t^3+1)}{3t^8-2t^7+2t^6-3t^5+t^4-3t^3-2t+1}$
386	$[1,3,2,2,3]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{3t^8-2t^7+2t^6-3t^5-3t^3-2t+1}$
387	$[1,3,2,2,4]$	$\frac{(t^2+t+1)(t^3+t^2+t+1)(t^3+1)}{2t^8-2t^7+t^6-3t^5-3t^3-2t+1}$
388	$[1,3,2,3,1]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{3t^8-2t^7+2t^6-2t^5-t^4-2t^3-2t+1}$
389	$[1,3,2,3,2]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{3t^8-2t^7+2t^6-3t^5-3t^3-2t+1}$
390	$[1,3,2,3,3]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{3t^8-2t^7+2t^6-3t^5-t^4-3t^3-2t+1}$

	$[r1, r2, r3, r4, r5]$	*Poincaré series*
391	$[1, 3, 2, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-t^4-3\,t^3-2\,t+1}$
392	$[1, 3, 2, 4, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{2\,t^8-2\,t^7+t^6-2\,t^5-t^4-2\,t^3-2\,t+1}$
393	$[1, 3, 2, 4, 2]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-3\,t^3-2\,t+1}$
394	$[1, 3, 2, 4, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-t^4-3\,t^3-2\,t+1}$
395	$[1, 3, 2, 4, 4]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{t^8-2\,t^7-3\,t^5-t^4-3\,t^3-2\,t+1}$
396	$[1, 3, 3, 1, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{3\,t^6-2\,t^5-t^4+t^3-t^2-2\,t+1}$
397	$[1, 3, 3, 1, 2]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{3\,t^8-2\,t^7+2\,t^6-2\,t^5-t^4-2\,t^3-2\,t+1}$
398	$[1, 3, 3, 1, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{3\,t^6-2\,t^5-t^4-t^2-2\,t+1}$
399	$[1, 3, 3, 1, 4]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{2\,t^6-2\,t^5-t^4-t^2-2\,t+1}$
400	$[1, 3, 3, 2, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{3\,t^8-2\,t^7+2\,t^6-2\,t^5-t^4-2\,t^3-2\,t+1}$
401	$[1, 3, 3, 2, 2]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{3\,t^8-2\,t^7+2\,t^6-3\,t^5-3\,t^3-2\,t+1}$
402	$[1, 3, 3, 2, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+2\,t^6-3\,t^5-t^4-3\,t^3-2\,t+1}$
403	$[1, 3, 3, 2, 4]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-t^4-3\,t^3-2\,t+1}$
404	$[1, 3, 3, 3, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{3\,t^6-2\,t^5-t^4-t^2-2\,t+1}$
405	$[1, 3, 3, 3, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+2\,t^6-3\,t^5-t^4-3\,t^3-2\,t+1}$
406	$[1, 3, 3, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{3\,t^6-2\,t^5-t^4-t^3-t^2-2\,t+1}$
407	$[1, 3, 3, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-2\,t^5-t^4-t^3-t^2-2\,t+1}$
408	$[1, 3, 3, 4, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{2\,t^6-2\,t^5-t^4-t^2-2\,t+1}$
409	$[1, 3, 3, 4, 2]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-t^4-3\,t^3-2\,t+1}$
410	$[1, 3, 3, 4, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-2\,t^5-t^4-t^3-t^2-2\,t+1}$

	$[r1, r2, r3, r4, r5]$	*Poincaré series*
411	$[1, 3, 3, 4, 4]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^6-2t^5-t^4-t^3-t^2-2t+1}$
412	$[1, 3, 4, 1, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{2t^6-2t^5-t^4+t^3-t^2-2t+1}$
413	$[1, 3, 4, 1, 2]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{2t^8-2t^7+t^6-2t^5-t^4-2t^3-2t+1}$
414	$[1, 3, 4, 1, 3]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{2t^6-2t^5-t^4-t^2-2t+1}$
415	$[1, 3, 4, 1, 4]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^6-2t^5-t^4-t^2-2t+1}$
416	$[1, 3, 4, 2, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{2t^8-2t^7+t^6-2t^5-t^4-2t^3-2t+1}$
417	$[1, 3, 4, 2, 2]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{2t^8-2t^7+t^6-3t^5-3t^3-2t+1}$
418	$[1, 3, 4, 2, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2t^8-2t^7+t^6-3t^5-t^4-3t^3-2t+1}$
419	$[1, 3, 4, 2, 4]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{t^8-2t^7-3t^5-t^4-3t^3-2t+1}$
420	$[1, 3, 4, 3, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2t^6-2t^5-t^4-t^2-2t+1}$
421	$[1, 3, 4, 3, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2t^8-2t^7+t^6-3t^5-t^4-3t^3-2t+1}$
422	$[1, 3, 4, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2t^6-2t^5-t^4-t^3-t^2-2t+1}$
423	$[1, 3, 4, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2t^5-t^4-t^3-t^2-2t+1}$
424	$[1, 3, 4, 4, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^6-2t^5-t^4-t^2-2t+1}$
425	$[1, 3, 4, 4, 2]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{t^8-2t^7-3t^5-t^4-3t^3-2t+1}$
426	$[1, 3, 4, 4, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2t^5-t^4-t^3-t^2-2t+1}$
427	$[1, 3, 4, 4, 4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{2t^5+t^4+t^3+t^2+2t-1}$
428	$[1, 4, 1, 4, 1]$	$\frac{\left(t^2+t+1\right)(t+1)^2}{t^4-t^3-3t^2-t+1}$
429	$[1, 4, 1, 4, 2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^5-2t^4-3t^3-2t^2-t+1}$
430	$[1, 4, 1, 4, 3]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^6-2t^5-t^4-t^2-2t+1}$

	$[r1, r2, r3, r4, r5]$	*Poincaré series*
431	$[1,4,1,4,4]$	$-\frac{\left(t^2+t+1\right)(t+1)^2}{2\,t^3+3\,t^2+t-1}$
432	$[1,4,2,2,1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2\,t^6-t^4-3\,t^3-2\,t^2-t+1}$
433	$[1,4,2,2,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2\,t^6-t^4-4\,t^3-2\,t^2-t+1}$
434	$[1,4,2,2,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-3\,t^3-2\,t+1}$
435	$[1,4,2,2,4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^5-2\,t^4-4\,t^3-2\,t^2-t+1}$
436	$[1,4,2,3,1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-2\,t^5-t^4-2\,t^3-2\,t+1}$
437	$[1,4,2,3,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-3\,t^3-2\,t+1}$
438	$[1,4,2,3,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-t^4-3\,t^3-2\,t+1}$
439	$[1,4,2,3,4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{t^8-2\,t^7-3\,t^5-t^4-3\,t^3-2\,t+1}$
440	$[1,4,2,4,1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^5-2\,t^4-3\,t^3-2\,t^2-t+1}$
441	$[1,4,2,4,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^5-2\,t^4-4\,t^3-2\,t^2-t+1}$
442	$[1,4,2,4,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{t^8-2\,t^7-3\,t^5-t^4-3\,t^3-2\,t+1}$
443	$[1,4,2,4,4]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2\,t^5+3\,t^4+4\,t^3+2\,t^2+t-1}$
444	$[1,4,3,2,1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-2\,t^5-t^4-2\,t^3-2\,t+1}$
445	$[1,4,3,2,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-3\,t^3-2\,t+1}$
446	$[1,4,3,2,3]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-t^4-3\,t^3-2\,t+1}$
447	$[1,4,3,2,4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^5+t^4+t^3+t^2+t+1\right)}{t^8-2\,t^7-3\,t^5-t^4-3\,t^3-2\,t+1}$
448	$[1,4,3,3,1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-2\,t^5-t^4-t^2-2\,t+1}$
449	$[1,4,3,3,2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-t^4-3\,t^3-2\,t+1}$
450	$[1,4,3,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-2\,t^5-t^4-t^3-t^2-2\,t+1}$

	$[r1, r2, r3, r4, r5]$	*Poincaré series*
451	$[1, 4, 3, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-t^4-t^3-t^2-2\,t+1}$
452	$[1, 4, 3, 4, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-t^4-t^2-2\,t+1}$
453	$[1, 4, 3, 4, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-3\,t^5-t^4-3\,t^3-2\,t+1}$
454	$[1, 4, 3, 4, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-t^4-t^3-t^2-2\,t+1}$
455	$[1, 4, 3, 4, 4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^5+t^4+t^3+t^2+2\,t-1}$
456	$[1, 4, 4, 1, 1]$	$\frac{\left(t^2+t+1\right)(t+1)^2}{t^4-t^3-3\,t^2-t+1}$
457	$[1, 4, 4, 1, 2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^5-2\,t^4-3\,t^3-2\,t^2-t+1}$
458	$[1, 4, 4, 1, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-t^4-t^2-2\,t+1}$
459	$[1, 4, 4, 1, 4]$	$-\frac{\left(t^2+t+1\right)(t+1)^2}{2\,t^3+3\,t^2+t-1}$
460	$[1, 4, 4, 2, 1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^5-2\,t^4-3\,t^3-2\,t^2-t+1}$
461	$[1, 4, 4, 2, 2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^5-2\,t^4-4\,t^3-2\,t^2-t+1}$
462	$[1, 4, 4, 2, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-3\,t^5-t^4-3\,t^3-2\,t+1}$
463	$[1, 4, 4, 2, 4]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2\,t^5+3\,t^4+4\,t^3+2\,t^2+t-1}$
464	$[1, 4, 4, 3, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-t^4-t^2-2\,t+1}$
465	$[1, 4, 4, 3, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-3\,t^5-t^4-3\,t^3-2\,t+1}$
466	$[1, 4, 4, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-t^4-t^3-t^2-2\,t+1}$
467	$[1, 4, 4, 3, 4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^5+t^4+t^3+t^2+2\,t-1}$
468	$[1, 4, 4, 4, 1]$	$-\frac{\left(t^2+t+1\right)(t+1)^2}{2\,t^3+3\,t^2+t-1}$
469	$[1, 4, 4, 4, 2]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2\,t^5+3\,t^4+4\,t^3+2\,t^2+t-1}$
470	$[1, 4, 4, 4, 3]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^5+t^4+t^3+t^2+2\,t-1}$

	$[r1, r2, r3, r4, r5]$	$Poincaré\ series$
471	$[1, 4, 4, 4, 4]$	$-\frac{\left(t^2+t+1\right)(t+1)^2}{t^4+3t^3+3t^2+t-1}$
472	$[2, 2, 2, 2, 1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{3t^6+t^5-4t^3-2t^2-t+1}$
473	$[2, 2, 2, 2, 2]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{3t^4-2t^3-t^2-2t+1}$
474	$[2, 2, 2, 2, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3t^8-2t^7+2t^6-4t^5+2t^4-4t^3-2t+1}$
475	$[2, 2, 2, 2, 4]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{2t^4-2t^3-t^2-2t+1}$
476	$[2, 2, 2, 3, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3t^8-2t^7+2t^6-3t^5+t^4-3t^3-2t+1}$
477	$[2, 2, 2, 3, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3t^8-2t^7+2t^6-4t^5+2t^4-4t^3-2t+1}$
478	$[2, 2, 2, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3t^8-2t^7+2t^6-4t^5+t^4-4t^3-2t+1}$
479	$[2, 2, 2, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2t^8-2t^7+t^6-4t^5+t^4-4t^3-2t+1}$
480	$[2, 2, 2, 4, 1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2t^6-t^4-4t^3-2t^2-t+1}$
481	$[2, 2, 2, 4, 2]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{2t^4-2t^3-t^2-2t+1}$
482	$[2, 2, 2, 4, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2t^8-2t^7+t^6-4t^5+t^4-4t^3-2t+1}$
483	$[2, 2, 2, 4, 4]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{t^4-2t^3-t^2-2t+1}$
484	$[2, 2, 3, 3, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3t^8-2t^7+2t^6-3t^5-3t^3-2t+1}$
485	$[2, 2, 3, 3, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3t^8-2t^7+2t^6-4t^5+t^4-4t^3-2t+1}$
486	$[2, 2, 3, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3t^8-2t^7+2t^6-4t^5-4t^3-2t+1}$
487	$[2, 2, 3, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2t^8-2t^7+t^6-4t^5-4t^3-2t+1}$
488	$[2, 2, 3, 4, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)\left(t^3+t^2+t+1\right)}{2t^8-2t^7+t^6-3t^5-3t^3-2t+1}$
489	$[2, 2, 3, 4, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2t^8-2t^7+t^6-4t^5+t^4-4t^3-2t+1}$
490	$[2, 2, 3, 4, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2t^8-2t^7+t^6-4t^5-4t^3-2t+1}$

	$[r1, r2, r3, r4, r5]$	*Poincaré series*
491	$[2, 2, 3, 4, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-4\,t^3-2\,t+1}$
492	$[2, 2, 4, 4, 1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^5-2\,t^4-4\,t^3-2\,t^2-t+1}$
493	$[2, 2, 4, 4, 2]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{t^4-2\,t^3-t^2-2\,t+1}$
494	$[2, 2, 4, 4, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-4\,t^3-2\,t+1}$
495	$[2, 2, 4, 4, 4]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)}{2\,t^3+t^2+2\,t-1}$
496	$[2, 3, 2, 3, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+2\,t^6-3\,t^5-3\,t^3-2\,t+1}$
497	$[2, 3, 2, 3, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+2\,t^6-4\,t^5+t^4-4\,t^3-2\,t+1}$
498	$[2, 3, 2, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+2\,t^6-4\,t^5-4\,t^3-2\,t+1}$
499	$[2, 3, 2, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-4\,t^5-4\,t^3-2\,t+1}$
500	$[2, 3, 2, 4, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-3\,t^3-2\,t+1}$
501	$[2, 3, 2, 4, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-4\,t^5+t^4-4\,t^3-2\,t+1}$
502	$[2, 3, 2, 4, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-4\,t^5-4\,t^3-2\,t+1}$
503	$[2, 3, 2, 4, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-4\,t^3-2\,t+1}$
504	$[2, 3, 3, 2, 1]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+2\,t^6-3\,t^5-3\,t^3-2\,t+1}$
505	$[2, 3, 3, 2, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+2\,t^6-4\,t^5+t^4-4\,t^3-2\,t+1}$
506	$[2, 3, 3, 2, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+2\,t^6-4\,t^5-4\,t^3-2\,t+1}$
507	$[2, 3, 3, 2, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-4\,t^5-4\,t^3-2\,t+1}$
508	$[2, 3, 3, 3, 1]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+2\,t^6-3\,t^5-t^4-3\,t^3-2\,t+1}$
509	$[2, 3, 3, 3, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+2\,t^6-4\,t^5-4\,t^3-2\,t+1}$
510	$[2, 3, 3, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+2\,t^6-4\,t^5-t^4-4\,t^3-2\,t+1}$

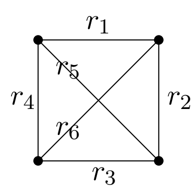
	$[r1, r2, r3, r4, r5]$	*Poincaré series*
511	$[2,3,3,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-4\,t^5-t^4-4\,t^3-2\,t+1}$
512	$[2,3,3,4,1]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-t^4-3\,t^3-2\,t+1}$
513	$[2,3,3,4,2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-4\,t^5-4\,t^3-2\,t+1}$
514	$[2,3,3,4,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-4\,t^5-t^4-4\,t^3-2\,t+1}$
515	$[2,3,3,4,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-t^4-4\,t^3-2\,t+1}$
516	$[2,3,4,2,1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-3\,t^3-2\,t+1}$
517	$[2,3,4,2,2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-4\,t^5+t^4-4\,t^3-2\,t+1}$
518	$[2,3,4,2,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-4\,t^5-4\,t^3-2\,t+1}$
519	$[2,3,4,2,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-4\,t^3-2\,t+1}$
520	$[2,3,4,3,1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-t^4-3\,t^3-2\,t+1}$
521	$[2,3,4,3,2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-4\,t^5-4\,t^3-2\,t+1}$
522	$[2,3,4,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-4\,t^5-t^4-4\,t^3-2\,t+1}$
523	$[2,3,4,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-t^4-4\,t^3-2\,t+1}$
524	$[2,3,4,4,1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{t^8-2\,t^7-3\,t^5-t^4-3\,t^3-2\,t+1}$
525	$[2,3,4,4,2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-4\,t^3-2\,t+1}$
526	$[2,3,4,4,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-t^4-4\,t^3-2\,t+1}$
527	$[2,3,4,4,4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^7+t^6+4\,t^5+t^4+4\,t^3+2\,t-1}$
528	$[2,4,2,4,1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)\left(t+1\right)}{t^6-t^5-2\,t^4-4\,t^3-2\,t^2-t+1}$
529	$[2,4,2,4,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t+1\right)}{t^4-2\,t^3-t^2-2\,t+1}$
530	$[2,4,2,4,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-4\,t^3-2\,t+1}$

	$[r1, r2, r3, r4, r5]$	*Poincaré series*
531	$[2, 4, 2, 4, 4]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)}{2\,t^3+t^2+2\,t-1}$
532	$[2, 4, 3, 3, 1]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-3\,t^5-t^4-3\,t^3-2\,t+1}$
533	$[2, 4, 3, 3, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-4\,t^5-4\,t^3-2\,t+1}$
534	$[2, 4, 3, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-4\,t^5-t^4-4\,t^3-2\,t+1}$
535	$[2, 4, 3, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-t^4-4\,t^3-2\,t+1}$
536	$[2, 4, 3, 4, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-3\,t^5-t^4-3\,t^3-2\,t+1}$
537	$[2, 4, 3, 4, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-4\,t^3-2\,t+1}$
538	$[2, 4, 3, 4, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-t^4-4\,t^3-2\,t+1}$
539	$[2, 4, 3, 4, 4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^7+t^6+4\,t^5+t^4+4\,t^3+2\,t-1}$
540	$[2, 4, 4, 2, 1]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^5-2\,t^4-4\,t^3-2\,t^2-t+1}$
541	$[2, 4, 4, 2, 2]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{t^4-2\,t^3-t^2-2\,t+1}$
542	$[2, 4, 4, 2, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-4\,t^3-2\,t+1}$
543	$[2, 4, 4, 2, 4]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)}{2\,t^3+t^2+2\,t-1}$
544	$[2, 4, 4, 3, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{t^8-2\,t^7-3\,t^5-t^4-3\,t^3-2\,t+1}$
545	$[2, 4, 4, 3, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-4\,t^3-2\,t+1}$
546	$[2, 4, 4, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-t^4-4\,t^3-2\,t+1}$
547	$[2, 4, 4, 3, 4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^7+t^6+4\,t^5+t^4+4\,t^3+2\,t-1}$
548	$[2, 4, 4, 4, 1]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2\,t^5+3\,t^4+4\,t^3+2\,t^2+t-1}$
549	$[2, 4, 4, 4, 2]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)}{2\,t^3+t^2+2\,t-1}$
550	$[2, 4, 4, 4, 3]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^7+t^6+4\,t^5+t^4+4\,t^3+2\,t-1}$

	$[r1, r2, r3, r4, r5]$	*Poincaré series*
551	$[2, 4, 4, 4, 4]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)}{t^4+2\,t^3+t^2+2\,t-1}$
552	$[3, 3, 3, 3, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{3\,t^6-2\,t^5-t^4-t^3-t^2-2\,t+1}$
553	$[3, 3, 3, 3, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+2\,t^6-4\,t^5-t^4-4\,t^3-2\,t+1}$
554	$[3, 3, 3, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{3\,t^6-2\,t^5-t^4-2\,t^3-t^2-2\,t+1}$
555	$[3, 3, 3, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-2\,t^5-t^4-2\,t^3-t^2-2\,t+1}$
556	$[3, 3, 3, 4, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{2\,t^6-2\,t^5-t^4-t^3-t^2-2\,t+1}$
557	$[3, 3, 3, 4, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7+t^6-4\,t^5-t^4-4\,t^3-2\,t+1}$
558	$[3, 3, 3, 4, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-2\,t^5-t^4-2\,t^3-t^2-2\,t+1}$
559	$[3, 3, 3, 4, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-t^4-2\,t^3-t^2-2\,t+1}$
560	$[3, 3, 4, 4, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^6-2\,t^5-t^4-t^3-t^2-2\,t+1}$
561	$[3, 3, 4, 4, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-t^4-4\,t^3-2\,t+1}$
562	$[3, 3, 4, 4, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-t^4-2\,t^3-t^2-2\,t+1}$
563	$[3, 3, 4, 4, 4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^5+t^4+2\,t^3+t^2+2\,t-1}$
564	$[3, 4, 3, 4, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^6-2\,t^5-t^4-t^3-t^2-2\,t+1}$
565	$[3, 4, 3, 4, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-t^4-4\,t^3-2\,t+1}$
566	$[3, 4, 3, 4, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-t^4-2\,t^3-t^2-2\,t+1}$
567	$[3, 4, 3, 4, 4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^5+t^4+2\,t^3+t^2+2\,t-1}$
568	$[3, 4, 4, 3, 1]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^6-2\,t^5-t^4-t^3-t^2-2\,t+1}$
569	$[3, 4, 4, 3, 2]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-4\,t^5-t^4-4\,t^3-2\,t+1}$
570	$[3, 4, 4, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-t^4-2\,t^3-t^2-2\,t+1}$

	$[r1, r2, r3, r4, r5, r6]$	*Poincaré series*
571	$[3, 4, 4, 3, 4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^5+t^4+2\,t^3+t^2+2\,t-1}$
572	$[3, 4, 4, 4, 1]$	$-\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{2\,t^5+t^4+t^3+t^2+2\,t-1}$
573	$[3, 4, 4, 4, 2]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^7+t^6+4\,t^5+t^4+4\,t^3+2\,t-1}$
574	$[3, 4, 4, 4, 3]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^5+t^4+2\,t^3+t^2+2\,t-1}$
575	$[3, 4, 4, 4, 4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6+2\,t^5+t^4+2\,t^3+t^2+2\,t-1}$
576	$[4, 4, 4, 4, 1]$	$-\frac{\left(t^2+t+1\right)(t+1)^2}{t^4+3\,t^3+3\,t^2+t-1}$
577	$[4, 4, 4, 4, 2]$	$-\frac{\left(t^3+t^2+t+1\right)(t+1)}{t^4+2\,t^3+t^2+2\,t-1}$
578	$[4, 4, 4, 4, 3]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6+2\,t^5+t^4+2\,t^3+t^2+2\,t-1}$
579	$[4, 4, 4, 4, 4]$	$-\frac{(t+1)^2}{2\,t^2+2\,t-1}$

The Poincaré series of Cartan matrices with reduced Coxeter graph 4-VI



	$[r1, r2, r3, r4, r5, r6]$	$Poincaré\, series$
580	$[1,1,1,1,1,1]$	$\frac{\left(t^2+t+1\right)(t+1)}{3\,t^3-2\,t^2-2\,t+1}$
581	$[1,1,1,1,1,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{3\,t^6+t^5-t^4-t^3-3\,t^2-t+1}$
582	$[1,1,1,1,1,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{3\,t^6-2\,t^5-2\,t^4+3\,t^3-2\,t^2-2\,t+1}$
583	$[1,1,1,1,1,4]$	$\frac{\left(t^2+t+1\right)(t+1)}{2\,t^3-2\,t^2-2\,t+1}$
584	$[1,1,1,1,2,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{3\,t^6+t^5-t^4-2\,t^3-3\,t^2-t+1}$
585	$[1,1,1,1,2,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+t^6-3\,t^4-t^2-2\,t+1}$
586	$[1,1,1,1,2,4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2\,t^6-2\,t^4-2\,t^3-3\,t^2-t+1}$
587	$[1,1,1,1,3,3]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{3\,t^6-2\,t^5-2\,t^4+2\,t^3-2\,t^2-2\,t+1}$
588	$[1,1,1,1,3,4]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{2\,t^6-2\,t^5-2\,t^4+2\,t^3-2\,t^2-2\,t+1}$
589	$[1,1,1,1,4,4]$	$\frac{t^2+t+1}{t^2-3\,t+1}$
590	$[1,1,1,2,2,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{3\,t^6+t^5-t^4-3\,t^3-3\,t^2-t+1}$
591	$[1,1,1,2,2,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+t^6-t^5-2\,t^4-t^3-t^2-2\,t+1}$
592	$[1,1,1,2,2,4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2\,t^6-2\,t^4-3\,t^3-3\,t^2-t+1}$
593	$[1,1,1,2,3,3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+t^6-t^5-3\,t^4-t^3-t^2-2\,t+1}$
594	$[1,1,1,2,3,4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7-t^5-3\,t^4-t^3-t^2-2\,t+1}$
595	$[1,1,1,2,4,4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^5-3\,t^4-3\,t^3-3\,t^2-t+1}$
596	$[1,1,1,3,3,3]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{3\,t^6-2\,t^5-2\,t^4+t^3-2\,t^2-2\,t+1}$
597	$[1,1,1,3,3,4]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{2\,t^6-2\,t^5-2\,t^4+t^3-2\,t^2-2\,t+1}$
598	$[1,1,1,3,4,4]$	$\frac{\left(t^2+t+1\right)\left(t^3+1\right)(t+1)}{t^6-2\,t^5-2\,t^4+t^3-2\,t^2-2\,t+1}$
599	$[1,1,1,4,4,4]$	$-\frac{\left(t^2+t+1\right)(t+1)}{2\,t^2+2\,t-1}$
600	$[1,1,2,2,2,2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{3\,t^6+t^5-t^4-4\,t^3-3\,t^2-t+1}$

	$[r1, r2, r3, r4, r5, r6]$	$Poincaré\, series$
601	$[1, 1, 2, 2, 2, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+t^6-2\,t^5-t^4-2\,t^3-t^2-2\,t+1}$
602	$[1, 1, 2, 2, 2, 4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2\,t^6-2\,t^4-4\,t^3-3\,t^2-t+1}$
603	$[1, 1, 2, 2, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+t^6-2\,t^5-2\,t^4-2\,t^3-t^2-2\,t+1}$
604	$[1, 1, 2, 2, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7-2\,t^5-2\,t^4-2\,t^3-t^2-2\,t+1}$
605	$[1, 1, 2, 2, 4, 4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^5-3\,t^4-4\,t^3-3\,t^2-t+1}$
606	$[1, 1, 2, 3, 3, 3]$	$\frac{\left(t^3+1\right)\left(t^3+2\,t^2+2\,t+1\right)\left(t^2+1\right)}{3\,t^8-2\,t^7+t^6-2\,t^5-3\,t^4-2\,t^3-t^2-2\,t+1}$
607	$[1, 1, 2, 3, 3, 4]$	$\frac{\left(t^3+1\right)\left(t^3+2\,t^2+2\,t+1\right)\left(t^2+1\right)}{2\,t^8-2\,t^7-2\,t^5-3\,t^4-2\,t^3-t^2-2\,t+1}$
608	$[1, 1, 2, 3, 4, 4]$	$\frac{\left(t^3+1\right)\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)}{t^8-2\,t^7-t^6-2\,t^5-3\,t^4-2\,t^3-t^2-2\,t+1}$
609	$[1, 1, 2, 4, 4, 4]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2\,t^5+4\,t^4+4\,t^3+3\,t^2+t-1}$
610	$[1, 1, 3, 3, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{3\,t^6-2\,t^5-2\,t^4-2\,t^2-2\,t+1}$
611	$[1, 1, 3, 3, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-2\,t^5-2\,t^4-2\,t^2-2\,t+1}$
612	$[1, 1, 3, 3, 4, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-2\,t^4-2\,t^2-2\,t+1}$
613	$[1, 1, 3, 4, 4, 4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^5+2\,t^4+2\,t^2+2\,t-1}$
614	$[1, 1, 4, 4, 4, 4]$	$-\frac{\left(t^2+t+1\right)(t+1)}{t^3+2\,t^2+2\,t-1}$
615	$[1, 2, 2, 2, 2, 2]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{3\,t^6+t^5-t^4-5\,t^3-3\,t^2-t+1}$
616	$[1, 2, 2, 2, 2, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+t^6-3\,t^5-3\,t^3-t^2-2\,t+1}$
617	$[1, 2, 2, 2, 2, 4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2\,t^6-2\,t^4-5\,t^3-3\,t^2-t+1}$
618	$[1, 2, 2, 2, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+t^6-3\,t^5-t^4-3\,t^3-t^2-2\,t+1}$
619	$[1, 2, 2, 2, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7-3\,t^5-t^4-3\,t^3-t^2-2\,t+1}$
620	$[1, 2, 2, 2, 4, 4]$	$\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6-t^5-3\,t^4-5\,t^3-3\,t^2-t+1}$

	$[r1, r2, r3, r4, r5, r6]$	$Poincaré\,series$
621	$[1, 2, 2, 3, 3, 3]$	$\frac{\left(t^3+1\right)\left(t^3+2\,t^2+2\,t+1\right)\left(t^2+1\right)}{3\,t^8-2\,t^7+t^6-3\,t^5-2\,t^4-3\,t^3-t^2-2\,t+1}$
622	$[1, 2, 2, 3, 3, 4]$	$\frac{\left(t^3+1\right)\left(t^3+2\,t^2+2\,t+1\right)\left(t^2+1\right)}{2\,t^8-2\,t^7-3\,t^5-2\,t^4-3\,t^3-t^2-2\,t+1}$
623	$[1, 2, 2, 3, 4, 4]$	$\frac{\left(t^3+1\right)\left(t^3+2\,t^2+2\,t+1\right)\left(t^2+1\right)}{t^8-2\,t^7-t^6-3\,t^5-2\,t^4-3\,t^3-t^2-2\,t+1}$
624	$[1, 2, 2, 4, 4, 4]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{2\,t^5+4\,t^4+5\,t^3+3\,t^2+t-1}$
625	$[1, 2, 3, 3, 3, 3]$	$\frac{\left(t^3+1\right)\left(t^3+2\,t^2+2\,t+1\right)\left(t^2+1\right)}{3\,t^8-2\,t^7+t^6-3\,t^5-3\,t^4-3\,t^3-t^2-2\,t+1}$
626	$[1, 2, 3, 3, 3, 4]$	$\frac{\left(t^3+1\right)\left(t^3+2\,t^2+2\,t+1\right)\left(t^2+1\right)}{2\,t^8-2\,t^7-3\,t^5-3\,t^4-3\,t^3-t^2-2\,t+1}$
627	$[1, 2, 3, 3, 4, 4]$	$\frac{\left(t^3+1\right)\left(t^3+2\,t^2+2\,t+1\right)\left(t^2+1\right)}{t^8-2\,t^7-t^6-3\,t^5-3\,t^4-3\,t^3-t^2-2\,t+1}$
628	$[1, 2, 3, 4, 4, 4]$	$-\frac{\left(t^2+t+1\right)\left(t^3+t^2+t+1\right)\left(t^3+1\right)}{2\,t^7+2\,t^6+3\,t^5+3\,t^4+3\,t^3+t^2+2\,t-1}$
629	$[1, 2, 4, 4, 4, 4]$	$-\frac{\left(t^3+t^2+t+1\right)\left(t^2+t+1\right)(t+1)}{t^6+3\,t^5+5\,t^4+5\,t^3+3\,t^2+t-1}$
630	$[1, 3, 3, 3, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{3\,t^6-2\,t^5-2\,t^4-t^3-2\,t^2-2\,t+1}$
631	$[1, 3, 3, 3, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^6-2\,t^5-2\,t^4-t^3-2\,t^2-2\,t+1}$
632	$[1, 3, 3, 3, 4, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6-2\,t^5-2\,t^4-t^3-2\,t^2-2\,t+1}$
633	$[1, 3, 3, 4, 4, 4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{2\,t^5+2\,t^4+t^3+2\,t^2+2\,t-1}$
634	$[1, 3, 4, 4, 4, 4]$	$-\frac{\left(t^5+t^4+t^3+t^2+t+1\right)(t+1)}{t^6+2\,t^5+2\,t^4+t^3+2\,t^2+2\,t-1}$
635	$[1, 4, 4, 4, 4, 4]$	$-\frac{\left(t^2+t+1\right)(t+1)}{2\,t^3+2\,t^2+2\,t-1}$
636	$[2, 2, 2, 2, 2, 2]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{3\,t^4-2\,t^3-2\,t^2-2\,t+1}$
637	$[2, 2, 2, 2, 2, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+t^6-4\,t^5+t^4-4\,t^3-t^2-2\,t+1}$
638	$[2, 2, 2, 2, 2, 4]$	$\frac{\left(t^3+t^2+t+1\right)(t+1)}{2\,t^4-2\,t^3-2\,t^2-2\,t+1}$
639	$[2, 2, 2, 2, 3, 3]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{3\,t^8-2\,t^7+t^6-4\,t^5-4\,t^3-t^2-2\,t+1}$
640	$[2, 2, 2, 2, 3, 4]$	$\frac{\left(t^5+t^4+t^3+t^2+t+1\right)\left(t^3+t^2+t+1\right)}{2\,t^8-2\,t^7-4\,t^5-4\,t^3-t^2-2\,t+1}$

	$[r1, r2, r3, r4, r5, r6]$	$Poincaré\,series$
641	$[2, 2, 2, 2, 4, 4]$	$\frac{(t^3+t^2+t+1)(t+1)}{t^4-2t^3-2t^2-2t+1}$
642	$[2, 2, 2, 3, 3, 3]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{3t^8-2t^7+t^6-4t^5-t^4-4t^3-t^2-2t+1}$
643	$[2, 2, 2, 3, 3, 4]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{2t^8-2t^7-4t^5-t^4-4t^3-t^2-2t+1}$
644	$[2, 2, 2, 3, 4, 4]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{t^8-2t^7-t^6-4t^5-t^4-4t^3-t^2-2t+1}$
645	$[2, 2, 2, 4, 4, 4]$	$-\frac{(t^3+t^2+t+1)(t+1)}{2t^3+2t^2+2t-1}$
646	$[2, 2, 3, 3, 3, 3]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{3t^8-2t^7+t^6-4t^5-2t^4-4t^3-t^2-2t+1}$
647	$[2, 2, 3, 3, 3, 4]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{2t^8-2t^7-4t^5-2t^4-4t^3-t^2-2t+1}$
648	$[2, 2, 3, 3, 4, 4]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{t^8-2t^7-t^6-4t^5-2t^4-4t^3-t^2-2t+1}$
649	$[2, 2, 3, 4, 4, 4]$	$-\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{2t^7+2t^6+4t^5+2t^4+4t^3+t^2+2t-1}$
650	$[2, 2, 4, 4, 4, 4]$	$-\frac{(t^3+t^2+t+1)(t+1)}{t^4+2t^3+2t^2+2t-1}$
651	$[2, 3, 3, 3, 3, 3]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{3t^8-2t^7+t^6-4t^5-3t^4-4t^3-t^2-2t+1}$
652	$[2, 3, 3, 3, 3, 4]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{2t^8-2t^7-4t^5-3t^4-4t^3-t^2-2t+1}$
653	$[2, 3, 3, 3, 4, 4]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{t^8-2t^7-t^6-4t^5-3t^4-4t^3-t^2-2t+1}$
654	$[2, 3, 3, 4, 4, 4]$	$-\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{2t^7+2t^6+4t^5+3t^4+4t^3+t^2+2t-1}$
655	$[2, 3, 4, 4, 4, 4]$	$-\frac{(t^5+t^4+t^3+t^2+t+1)(t^3+t^2+t+1)}{t^8+2t^7+3t^6+4t^5+3t^4+4t^3+t^2+2t-1}$
656	$[2, 4, 4, 4, 4, 4]$	$-\frac{(t^3+t^2+t+1)(t+1)}{2t^4+2t^3+2t^2+2t-1}$
657	$[3, 3, 3, 3, 3, 3]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)}{3t^6-2t^5-2t^4-2t^3-2t^2-2t+1}$
658	$[3, 3, 3, 3, 3, 4]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)}{2t^6-2t^5-2t^4-2t^3-2t^2-2t+1}$
659	$[3, 3, 3, 3, 4, 4]$	$\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)}{t^6-2t^5-2t^4-2t^3-2t^2-2t+1}$
660	$[3, 3, 3, 4, 4, 4]$	$-\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)}{2t^5+2t^4+2t^3+2t^2+2t-1}$
661	$[3, 3, 4, 4, 4, 4]$	$-\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)}{t^6+2t^5+2t^4+2t^3+2t^2+2t-1}$
662	$[3, 4, 4, 4, 4, 4]$	$-\frac{(t^5+t^4+t^3+t^2+t+1)(t+1)}{2t^6+2t^5+2t^4+2t^3+2t^2+2t-1}$
663	$[4, 4, 4, 4, 4, 4]$	$-\frac{t+1}{3t-1}$